Homework 2
Routine Practice Exercises Not to Be Turned In

1. The Ramsey number $R(k,l)$ is the smallest n so that every two-coloring (blue and red) of the complete graph on n vertices contains either a monochromatic (blue) complete graph with k vertices or a monochromatic (red) complete graph with l vertices. Show that if there exists p between 0 and 1 so that

$$\binom{n}{k} p^{\binom{k}{2}} + \binom{n}{l} (1-p)^{\binom{l}{2}} < 1$$

then $R(k,l) > n$.

Exercises to Turn In. Due Date: Friday, January 28

1. Ross 1.12

2. Ross 1.36 (Look up Jensen's inequality in the text on your own.)

3. Ross 1.43 (Formulate a more precise and correct version of this problem and then prove it.)

4. Ross 1.9

5. (The Kraft Inequality) Let F be a finite collection of binary strings of finite length and assume that no string in F is a prefix to any other string in F. For instance, the string 0 is a prefix to the string 00 and also to the string 01 but the string 1 is not a prefix to 00 or to 01. Let N_i denote the number of strings in F of length i. Can you give a probabilistic argument to show that

$$\sum_i \frac{N_i}{2^i} \leq 1?$$

Hint: If you write down 0's and 1's at random, stopping when you hit an element of F, what is your probability of hitting an element in F?