Homework 12

Exercises to Turn In. Due Date: Friday, April 29.

1. Ross 8.1. Use our definition of BM as a process that satisfies four conditions: (i) $X(t) \sim N(0,t)$, (ii) $X(t)$ has independent increments, (iii) $X(t)$ has stationary increments, (iv) $X(t)$ is continuous. For part (d), consider what would happen if $P(T = 0) < 1$. Then there is some positive probability that $T > \epsilon$ for some $\epsilon > 0$. See if you can get a contradiction using the fact that $X(t)$ and $Y(t)$ are Brownian motion.

2. Ross 8.21.

3. From Russ Lyon's Lecture Notes: Let $X(t)$ be Brownian motion with drift and let $Y(t)$ be the martingale $X(t) - \mu t$. Let T be the stopping time that stops the process when it hits A or $-B$. Use this martingale to calculate ET. What is ET for standard Brownian motion ($\mu = 0$)? Hint: Use Section 8.4 (also covered in class).

4. From Rick Bradley's problems: Consider a geometric Brownian motion: $Y(t) = 100e^{X(t)}$ where $X(t)$ is standard Brownian motion, $Y(0) = \$100$ models the value of a stock at the present time, and t measures time in years. What is the probability that the stock will reach \$200 sometime during the next two years? (Hint: use the Reflection Principle.)

5. Ross 8.23.