Homework 11

Exercises to Turn In. Due Date: Friday, April 22.

1. (a) Ross 6.4.

   (b) Adapted from Rick Bradley's notes: Consider the previous problem and assume $p \neq 1/2$. Let $Z_n = (q/p)^{S_n}$ as above. Let τ be the first time S_n reaches A or $-B$ ($A > 0$, $B > 0$). Use EZ_τ to compute the probability the random walk S_n reaches A before it reaches $-B$.

2. Ross 6.9. Write Z_n in terms of Z_{n+1} and X_{n+1} and use ideas from class about conditioning on more information.

3. Ross 6.10. As discussed in class. Also see example 6.2(a).

4. Ross 6.11. Hint: Ideas can be found in example 6.2(c). Show that $Z_n = (A - X_n)(B + X_n) + n$ is a martingale.

5. From Russ Lyon's Lecture Notes: Let X be 2 with probability $p > 1/3$ and let $X = -1$ with probability $1 - p$. Let S_n be the corresponding random walk: $S_n = \sum_{i=1}^n X_i$. Let N be the first time that the random walk is positive. What is the distribution of S_N? Hint: what values can S_N take on? Pick one of them and use a first step analysis to find its probability. Find ES_N and EN.