Homework 10

Exercises to Turn In. Due Date: Friday, April 15.

1. Adapted from Russ Lyon's Lecture Notes: Consider a particle that can be of two different types, type 1 and type 2. At exponential times (with rate λ) the particle chooses a type. It chooses to be type 1 with probability α_1 and it chooses to be type 2 with probability α_2. Suppose that the distribution at time zero is that the particle is type 1 with probability p_1 and type 2 with probability p_2 so that $p_1 + p_2 = 1$. Now consider the continuous time Markov process for which the states are the type of the particle at time t. What is the 2×2 rate matrix, Q, for the continuous-time process? What is the probability that the process is in state 1 at time t?

2. Ross 5.14. Hint: The answer in the back of the text is a bit off given the way 5.14 is written, but it gives the right idea.

3. Ross 5.20. Hint: You can simplify the birth and death process formulas a little, but I don't get a simple numeric answer when I work this problem.

4. Ross 5.21. Hint: There is one chair for waiting and one chair for service in the barber shop. If a customer passes by when two people are in the shop already, the customer departs.

5. From Rick Bradley's problem sets: Let $X(t)$ be the Yule process (see p. 235 in Ross) with $X(0) = 1$. Show that for all $\epsilon > 0$,
$$\lim_{t\to\infty} P\left(\lambda - \epsilon < \frac{\ln X(t)}{t} < \lambda + \epsilon\right) = 1$$
so that the asymptotic growth of the Yule process is roughly exponential. Hints: Show that both tails are smaller than $e^{-\epsilon t}$. Use Markov's inequality for $P(X(t) > e^{(\lambda+\epsilon)t})$. Use the distribution of $P_{1,j}(t)$ to bound $P(X(t) < e^{(\lambda-\epsilon)t})$. Don't sum the geometric series. Instead, replace $\left(1 - e^{-\lambda t}\right)^{j-1}$ by the gross bound of 1.