

Study Guide for the Final Exam

The exam will consist of 10 questions. Your best 7 out of 10 problems will be graded on a 15-point scale. The score will be out of 100. That is, if you work 7 problems perfectly, you will have 5 points extra credit. Six problems will be over new material. Four problems will come from the past exams. That is, if you understand the material on the previous exams and write your answers to those perfectly on the final, you will have earned 60/100 or a B-. If, in addition, you can work 2 problems over the new material, you will have earned an A on the final exam. Of course, working any 6-7 problems correctly will earn you an A on the final.

1. Be able to calculate $P(t)$, the transition matrix at time t , for a continuous Markov chain. For specific calculations, see the 2-d case as done several times in the lectures and as Problem #1 on Homework 10. Whatever the dimension, transition matrices for the uniformized process with all rows equal provide relatively simple, computable examples. Why?
2. Know Kolmogorov's Backward Equation. If you use my version of the rate matrix Q , the formula is quite simple in terms of matrix algebra. Be able to show that a process satisfies the Kolmogorov Backwards equation. You showed the Yule process satisfies it your homework. Show that the 2-d examples in the previous item satisfy it. Be able to show that the transition density for Brownian motion satisfies the Backward Diffusion Equation.
3. Be able to find and use the stationary distribution for a continuous time Markov chain or a semi-Markov process. Examples of this include:
 - (a) Homework 9 # 2
 - (b) Homework 10 # 4
4. There will definitely be at least one problem on the exam that can be approached efficiently using a first step analysis. The most recent problem in your homework that was effectively approached with a first step analysis was Homework 11 # 5.
5. Be able to use the Martingale Stopping Theorem to calculate expected times to events and probabilities. Examples include:
 - (a) Homework 11 # 1
 - (b) Homework 11 # 4
 - (c) Homework 12 # 3
6. Brownian motion: study Homework 12.