Study Guide for Exam 1

1. Be able to use a "first step analysis" (conditioning on the outcome of a first step) to find the recurrence relation describing some probability. The Ballot Problem is one example. There, the best "first step" was to condition on the outcome of the last voter. But this sort of thing occurs quite often. If you have $n and I have $m and we play a game where I give you $1 if a coin toss is heads and you give me $1 if the toss is tails, then we can analyze the probability I end up with all of your money using a first step analysis as well (that's called Gambler's ruin). Another problem that can be analyzed using recurrences is the following: Urn A contains 3 white balls and one blue ball. Urn B contains 4 white balls. Choose a ball at random from each urn and then place those balls at random one into each urn (so both urns always have 4 balls in them.) What is the probability urn A contains the blue ball after k iterations of this process? You could also use a first step analysis to compute the expected number of iterations needed to first move the blue ball to urn B. This is actually a geometric distribution, but a first step analysis should lead you to the correct (same) value.

2. Be able to apply the probabilistic method to a situation you have already seen or with assistance. Study all homework problems and the examples used in class.

3. Be able to show that the interarrival distribution of a Poisson process is exponential and that the number of events of an exponentially-distributed interarrival time process occurring before time t is Poisson.

4. For a homogeneous Poisson process, the local rate is constant. For an inhomogeneous Poisson process, the local rate may depend on where you are. Suppose that rate is $\lambda(t) = \lambda \times t$. What does that say about the Poisson process? What is the mean number of events in the interval $(0, 1)$, $(1, 2)$, $(100, 101)$? If you wanted to transform time so that the process was homogeneous, how would you do it? That is, choose a function f so that if $s = f(t)$, then $N(s)$ is a homogeneous Poisson process with rate λ (or at least with a constant rate).

5. Be able to show that $EX = \int_0^\infty (1 - F(x))dx$ for non-negative random variables X rigorously. Be able to apply this formula to get a expectation.

6. Understand what "infinitely often" means and be able to use the Borel-Cantelli Lemmas to show that something does or does not happen infinitely often (with probability 1).