Math 212 Practice Final Exam

1. Evaluate $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

2. Evaluate $\arcsin\left(\sin\frac{7\pi}{6}\right)$

3. Evaluate $\sec\left(\arctan\frac{4}{3}\right)$

4. Find the derivative of $y = \arcsin 2\sqrt{x}$.

5. Find the derivative of $y = x\sec^{-1}(3x)$.

6. Write out the FORM for the partial fraction decomposition of the integral below. Do not evaluate the coefficients:

$$\int \frac{(x^6-1)dx}{(x-1)(x+2)^2(x^2+5)^2}$$

7. If you were to use a trig substitution to remove the radical in the integral below, what would it be?

$$\int \frac{dx}{\sqrt{4x^2+9}}$$

8. Evaluate $\int \cos^2 axdx$ where a is a constant.

9. Evaluate $\frac{x^3dx}{x-2}$.

10. Evaluate $\int \arctan x\, dx$

11. Evaluate $\int \frac{x\, dx}{\sqrt{1+x^2}}$.

12. Evaluate $\int \frac{3x^2-4x+3}{x^3+x}\, dx$

13. Evaluate $\int \frac{dx}{x^2\sqrt{x^2-1}}$.

14. Determine whether each integral is convergent or divergent. Evaluate the convergent ones.

a. $\int_1^\infty \frac{dx}{1+x^2}$ b. $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}$ c. $\int_0^4 \frac{dx}{x-3}$

15. Set up, but do not evaluate, the integral for the volume of the solid obtained by rotating the area enclosed by $y = e^x$, $x = 0$, $y = 0$, and the line $x = 2$ about the y-axis.

16. Set up, but do not evaluate, the integral for the volume of the solid obtained by rotating the area enclosed by $y = e^x$, $x = 0$, $y = 0$, and the line $x = 2$ about the line $x = 4$.

17. Set up, but do not evaluate, the integral for the surface area generating by revolving the arc given by $y = 1 + \sin x$ from $x = 0$ to $x = \pi/2$ about the x-axis.

18. Set up, but do not evaluate, the integral for the surface area generating by revolving the arc given by $y = 1 + \sin x$ from $x = 0$ to $x = \pi/2$ about the y-axis.

19. A fish tank has a rectangular base of 2 ft. and a length of 4 ft, and rectangular sides of height 2 ft. If the tank is half-filled with water, find the work required to pump all the water out of the tank through a pipe that extends 1 ft above the top of the tank. (Water weighs 62.5 lbs/ft^3.)

20. Find the length of the arc of the curve $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ from $x = 1$ to $x = 2$.

21. A solid is generated by revolving the area bounded by $y = 1/x$, $x = 1$, and $y = 0$ about the x-axis. Does this solid have finite volume? If so, what is the volume? Does it have finite surface area? If so, find the surface area.

22. Find an expression for the nth term in the sequence

$$\frac{1}{e}, \frac{8}{e^2}, \frac{27}{e^3}, \frac{64}{e^4}, \ldots$$

23. Find an expression for the nth term in the sequence

$$\frac{1}{3}, -\frac{2}{5}, \frac{3}{7}, -\frac{4}{9}, \frac{5}{11}, \ldots$$

24. An infinite series has the property that $2a_n + 5a_{n+1} = 0$ for all $n \geq 1$. If the sum of the series is $\frac{25}{7}$, find the first 5 terms of the series.

25. For what values of x does the series $\sum_{n=1}^{\infty} \frac{n^2}{n^{3x}}$ converge?

26. For what values of x does the series $\sum_{n=1}^{\infty} \left(\frac{6}{x}\right)^n$ converge?

27. Given the series $\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$, find the first 5 parial sums, $S_1, S_2, \ldots, S_5$. Find an expression for S_n. Use S_n to determine whether or not the series converges. If it does converge, give the sum. If it does not converge, show why.

28. True or false:

a) Every increasing sequence is unbounded.

b) If $0 \leq a_n \leq b_n$ and $\sum a_n$ diverges, then so does $\sum b_n$.

c) $\sum_{n=1}^{\infty} \frac{1}{n^{11/12}}$ diverges.

d) If $S_n = \frac{3n^2 + 5n + 1}{2 + 6n^2}$ represents the nth partial sum of $\sum a_i$, then the sum of the series is $\frac{1}{2}$.

e) If $|x| < 1$, then $\sum_{n=1}^{\infty} x^n = \frac{x}{1-x}$.

f) $\sum_{n=1}^{\infty} (-1)^n \frac{10}{n}$ converges conditionally.

g) If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges

h) The root test can be applied to an alternating series.

29. Use an appropriate test to show that the series converges absolutely, converges conditionally, or diverges.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+n+1}{n^4+2}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n+4n^4}}$

c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n3^n}{n!}$

d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1+n}{1+2n}\right)^n$

f) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$