Practice Exam 1

On Exam 1, there will be six problems. Your best five out of those will be graded. If you work all 6 correctly, that will be noted.

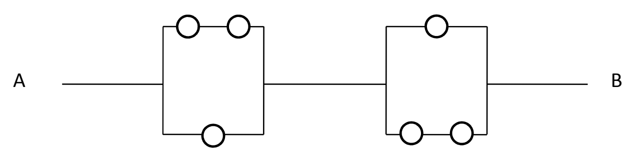
1. (a) Recall that $\binom{n}{k} = \binom{n}{n-k}$. Use a combinatorial argument to prove

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}$$

(b) Use a combinatorial argument to prove $\binom{n+1}{k+1} = \sum_{m=k}^{n} \binom{m}{k}$.

2. How many words (loosely defined as arrangements of letters) can be found by re-arranging all the letters in CONNECTICUT?

3. In the following diagram, circles represent switches that fail with probability p independent of each other. What is the probability that current flows from A to B?



4. (a) Prove or find a (concrete) counterexample: if A and B are independent then A^c and B^c are independent.
(b) If A and B are disjoint, can they be independent?

5. Polygraph tests are notoriously bad at catching liars and they indicate a substantial portion of truth-tellers are liars. The false positive rate for polygraph tests is about 16% and the false negative rate is about 20%. Suppose a company engaged in sensitive government work routinely screens all its applicants. Assume that the majority of its applicants (95%) tell the truth (knowing that they are going to have to take a polygraph test) but that about 5% lie (in part, knowing that there is a reasonable chance they will get away with the lies). What is the probability a randomly screened applicant is really lying given that the polygraph indicates he is lying?

6. Let $f(x) = cx^2$ for $0 \le x \le 1$ and let f be zero otherwise. For what value of c is f a probability density function? How would you generate a random sample from this distribution given uniform random numbers in $[0, 1]$?