Math M463/S463 Combinatorial Proof Practice

1. Use a counting argument to prove:
$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$

2. Use a counting argument to prove:
$$\sum_{k=0}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$$

3. Use a counting argument to prove:
$$\binom{2n}{n} = 2\binom{n}{2} + n^2$$

4. Use a counting argument to prove:
$$n^3 = 1 + 3(n-1) + 3(n-1)^2 + (n-1)^3$$

5. Use a counting argument to prove:
$$\sum_{k=m}^{n} \binom{k}{m} = \binom{n+1}{m+1}$$

6. Use a counting argument to prove:
$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$