Final Exam

Name: _____

1. Argue that the sum of two independent Binomial random variables, one with n trials and one with m trials, but with the same probability of success, p, has to be Binomially distributed due to what Binomial random variables are and what they model.

2. Argue that the sum of two independent Binomial random variables with the same probability of success is Binomial finding the density function of the sum from the density functions of the two components.

3. Argue that the sum of two independent Binomial random variables with the same probability of success has to be Binomial using moment generating functions. That is, find the moment generating function of a Binomial random variable and use the facts that the moment generating function for the sum is the product of the moment generating functions for the components and that moment generating functions determine the distribution when they exist in a neighborhood of zero.

4. Drug tests work differently than lie detector tests. For marijuana testing, a common test gives false positives with probability 0.02 approximately but false negatives with probability 0.40 approximately. Assume approximately 20% of high school seniors have used marijuana recently. What is the probability that a student is clean (has not used marijuana, at least not recently) given that the test for marijuana use is negative?

5. Let $X_n \to c$ in probability where c is a constant. Assume that g is a continuous function at c. Show that $g(X_n) \to g(c)$ in probability.

6. Let P be uniform on $[0, \frac{1}{2}]$. Given $P = p$, let X be Bernoulli with parameter p. Find the conditional distribution of P given X.

7. State the Central Limit Theorem.

8. Use the delta method to find the approximate mean and variance of $Y = \sqrt{X}$ where X is uniform on $[0, 2]$.

9. Find the distribution of $Y = \sqrt{X}$ where X is uniform on $[0, 2]$ exactly and calculate its mean and the variance.

10. Give an example of two uncorrelated random variables which are not independent.