Practice Final Exam

1. Argue that the sum of two independent Poisson random variables has to be Poisson with the sum of the two rates due to what Poisson random variables are and what they model.

2. Argue that the sum of two independent Poisson random variables has to be Poisson with the sum of the two rates by finding the density function of the sum from the density functions of the two components.

3. Argue that the sum of two independent Poisson random variables has to be Poisson with the sum of the two rates using moment generating functions. That is, find the moment generating function of a Poisson random variable and use the facts that the moment generating function for the sum is the product of the moment generating functions for the components and that moment generating functions determine the distribution when they exist in a neighborhood of zero.

4. Lie detector tests are notoriously flawed. Assume that a particular test gives false positives with probability 0.2 and false negatives with probability 0.1. In a large workforce assume that 15% of applicants lie. What is the probability that if the lie detector test says a random applicant lies that the applicant really is lying.

5. Let $X_n \to c$ in probability where c is a constant. Assume that g is a continuous function at c. Show that $g(X_n) \to g(c)$ in probability.

6. Let P be uniform on $[1/2, 1]$. Given $P = p$, let X be Bernoulli with parameter p. Find the conditional distribution of P given X.

7. Let $U_1, U_2, \ldots, U_{20}$ be i.i.d. uniform random variables on $[0, 1]$. Find the joint density function for the fourth, fifteenth, and twentieth order statistics.

8. Use the delta method to find the approximate mean and variance of $Y = \sqrt{X}$ where X is exponential with rate 1.

9. Find the distribution of $Y = \sqrt{X}$ where X is exponential with rate 1 exactly and calculate its mean and the variance. Is the approximation found in Problem 8 good? You may use the following facts:

$$\int_0^\infty t^2 e^{-t^2} dt = \frac{\sqrt{\pi}}{4} \qquad \text{and} \qquad \int_0^\infty t^3 e^{-t^2} dt = \frac{1}{2}$$

10. Give an example of two uncorrelated random variables which are not independent.