Practice Exam 2

1. Consider the following joint density for two discrete random variables:

	Y		
X	0	1	2
0	.1	.2	.1
1	.2	.2	.2

Find:
A: The marginal distributions $f_X(x)$ and $f_Y(y)$.
B: The conditional distributions $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.
C: EX, EY, $\text{Var}(X)$, and $\text{Var}(Y)$.
D: The correlation between X and Y.
E: The distribution of $Z = X + Y$.
F: The joint distribution of $U = X + Y$ and $V = X - Y$.

2. Consider the following joint density for two continuous random variables:

$$f_{X,Y}(x, y) = e^{-y} \text{ for } 0 < x < y < \infty$$

Find:
A: The marginal distributions $f_X(x)$ and $f_Y(y)$.
B: The conditional distributions $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.
C: EX, EY, $\text{Var}(X)$, and $\text{Var}(Y)$.
D: The correlation between X and Y.
E: The distribution of $Z = X + Y$.
F: The joint distribution of $U = X + Y$ and $V = X - Y$.

3. Find the joint distribution of the second ($X_{(2)}$) and the last ($X_{(n)}$) order statistics for n i.i.d. random variables if their marginal distributions are given by the density $f_X(x)$ and their individual cumulative distributions are given by $F_X(x)$.

4. Let $f(x) = 6x^2(1-x)^2$ for $-1 \leq x \leq 1$. Describe an algorithm to generate random variables from this distribution using the rejection method. In what proportion of trials will you accept the generated random variable?

5. State Markov's inequality and give an example of a random variable where the inequality is an equality.