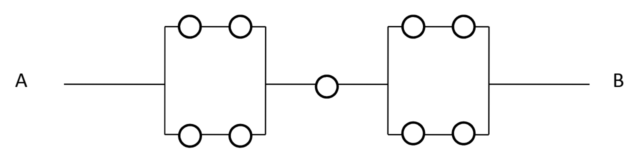
Exam 1

Name: ______________________________

Your best 5 out of 6 problems will form the basis of your grade on this exam. If you work all 6 problems essentially correctly, that will also be noted.

1. Let $f(x) = c/x^3$ for $x > 1$ and let f be zero otherwise. For what value of c is f a probability density function? How would you use uniform $[0, 1]$ random numbers to generate random numbers from this distribution?

2. In the following diagram, circles represent switches that fail with probability p independent of each other. What is the probability that current flows from A to B?



3. Suppose that the prevalence of prostate cancer in a certain male population is 30% and that high levels of prostate specific antigen (PSA) in the blood is used as a screen for prostate cancer. Approximately 20% of men with prostate cancer have normal PSA levels and approximately two thirds of men with high PSA levels do not have cancer. Use these numbers to determine the probability that a random man who has a high PSA levels actually has prostate cancer.

4. Let Z be a standard normal random variable with density

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

Find the density for the random variable $X = Z^2$.

5. Ten distinct (they unlock different doors) symmetrical keys are placed on a circular key ring. In how many ways can this be done?

6. Use a combinatorial argument to prove

$$\sum_{k=0}^{n} k \binom{n}{k} = n2^{n-1}$$