Practice Exam 1

1: State Arrow's Theorem (completely, giving all hypotheses as well as the conclusion.)

For problems 2-5 refer to the following table of voter preferences.

	Voters						
	A	B	C	D	E	F	G
ranking	Y	Y	Y	X	X	Z	Z
of	X	X	Z	Z	Z	Y	Y
candidates	Z	Z	X	Y	Y	X	X

2: Who is the winner of the election using the plurality method? What happens if candidate X drops out (or is moved to the last spot of everyone's preference list?) Is there a contradiction to an axiom from Arrow's theorem? If so, which one?

3: Who is the winner according to the Borda count? What happens again if candidate X drops out (or is moved to the last spot of everyone's preference list?) Is there a contradiction to an axiom from Arrow's theorem? If so, which one?

4: What is the societal ranking of candidates determined by the pairwise comparisons and majority rule method? Does this contradict an axiom to Arrow's theorem? Which one?

5: Suppose you know the voter preferences given in the table above and suppose you want candidate Y to win. Arrange a sequential voting method to guarantee the outcome you desire.

6: Flow chart the method and write Maple code to form a random sum in the following manner: for each integer i from 1 to 100, include i in the sum with probability .5. What is the average sum that results from this process? You should be able to answer this question analytically and therefore check that your code works.

7: Explain why the recurrence relation for the probability you win all of your opponent's money in a fair gambler's ruin set up (you have \$$x$ in your pocket and your opponent has \$$y$, you flip a fair coin exchanging dollar bills until one of you is broke) is

$$P_x = \frac{1}{2}P_{x+1} + \frac{1}{2}P_{x-1}$$

where P_x is the probability you win everything given that you start with \$$x$ and the total amount of money is kept constant at \$$(x+y)$. What are the boundary conditions? That is, what is P_0 and P_{x+y}? The resulting equation is like a second derivative equals zero situation so a guess at the solution is $P_x = Ax + B$. Show that such a P_x solves the recurrence relation and find A and B. Show that $P_x = \frac{x}{x+y}$.

8: Genotypic frequencies are tracked in a population. At the beginning of each generation, the population divides itself into two groups randomly but not evenly. Group 1 constitutes 20% of the population and reproduces by selfing. Group 2 constitutes the other 80% of the population and reproduces randomly. The initial genotypic frequencies for a gene with 2 alleles, A and a, are:

AA	Aa	aa
.2	.5	.3

What are the genotypic frequencies one generation later?

9: Assume the first 10 random numbers generated for u would be
0.3957188605
0.1931398164
0.02242417046
0.8001874845
0.4275520569
0.8426226844
0.4122862858
0.9964172142
0.3864083074
0.6946071893
What was the output from the following MAPLE code?

```
x := 0:
while (x < 100) do
u := rand()/10^12;
if (u < .3) then x := x + 30 elif (u < .6) then x := 2*x else x := x - 10 end if;
end do:
print(x);
```

10: For the discrete population growth model $x_{n+1} = 2(x_n - 10)$ find the fixed point and discuss the convergence for starting values away from this fixed point.