Lecture 13

Making Comparisons after ANOVA

In this lecture we will discuss how to make comparisons (contrasts) after you have run an ANOVA. It is best if these contrasts are planned before your run the ANOVA and the ANOVA is just contributing to a pre-planned comparison.

Comparing two populations after an ANOVA

Suppose you have run an ANOVA on data from I populations. You want to compare the means of populations i and j afterwards. Then you do a pooled-variance t-test, or form a pooled-variance confidence interval, pretty much as usual EXCEPT that you use the common standard deviation obtained from the ANOVA and you get all the degrees of freedom associated with that common standard deviation. The formula for the confidence interval for the difference in the two means then is

$$\left(\bar{Y}_{i,\cdot} - \bar{Y}_{j,\cdot}\right) \pm t_{\text{Error d.f.},\alpha/2} s_{\text{pooled from ANOVA}} \sqrt{\frac{1}{n_i} + \frac{1}{n_j}}$$

Example 1 Form the 95% confidence interval for the difference in the percent women on the venires of Judge A and the trial judge in Dr. Spock's conspiracy case.

From the ANOVA, we get $s = 7.007$ with 39 degrees of freedom. From using Minitab under Calc > Probability Distributions > t.. and asking for the Inverse Cumulative Distribution value for $\alpha/2 = 0.025$ with 39 degrees of freedom, we find $t = 2.02269$. Also from the ANOVA output we find $\bar{Y}_A = 34.2$ with $n_A = 5$ ad $\bar{Y}_{\text{Trial Judge}} = 15$ with $n_{\text{Trial Judge}} = 9$.

Thus the confidence interval is

$$(34.2 - 15) \pm (2.02269)(7.007)\sqrt{\frac{1}{5} + \frac{1}{9}} = (11.3, 27.1)$$

The we are 95% confident that the percent women Judge A has on his venires is between 11.3 and 27.1 more than the trial judge's percent women on his venires. ■

Contrasts

A contrast should be a planned comparison but it might involve all or many of the populations. For instance, for the percent women on the venires for various judges and for Spock's trial judge, on might know in advance one wants to compare the mean percent women on the venires for Spock's trial judge to some collective average for the other judges. We already did this one way, by asking if all the other judge's had similar percent women on their venires, determining that they did, pooling the data for those judges, and then contrasting the trial judge's percents to the collection of percents from the other judges.

The mathematical formulation of a contrast is thus:

$$\gamma = c_1\mu_1 + c_2\mu_2 + \cdots + c_I\mu_I$$

is the theoretical population contrast value we want information about. It is estimated using the sample means as

$$g = c_1\bar{Y}_{1,\cdot} + c_2\bar{Y}_{2,\cdot} + \cdots + c_I\bar{Y}_{I,\cdot}$$

and this statistic has standard error

$$\text{S.E.}(g) = s\sqrt{\frac{c_1^2}{n_1} + \frac{c_2^2}{n_2} + \cdots + \frac{c_I^2}{n_I}}$$

Generally, $c_1 + c_2 + \cdots + c_I = 0$ to provide a true contrast. However, this is not essential to the theory. Also note that the comparison we made above between the trial judge and judge A is a special contrast where $c_B = c_C = c_D = c_E = c_F = 0$.

Example 2 Consider the following contrast for the percent women on the venires of various judges:

$$\gamma = \mu_{\text{Trial Judge}} - \frac{\mu_A + \mu_B + \mu_C + \mu_D + \mu_E + \mu_F}{6}$$

The question of whether the trial judge had an unusually small percentage of women on his venires or not translates into the question of whether γ is less than zero or not.

As test of this hypothesis involves forming the contrast based on the sample data, finding its standard error, and computing the p-value for the resulting t-test:

$$g = 15 - \frac{34.2 + 33.5 + 29 + 27 + 28.143 + 26.222}{6} = -14.6775$$

$$\text{S.E.}(g) = (7.007) * \sqrt{\frac{1}{9} + \left(\frac{1}{6}\right)^2 \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{2} + \frac{1}{7} + \frac{1}{9}\right)} = 2.70291$$

$$t = \frac{g - 0}{\text{S.E.}(g)} = -5.43026$$

Using the probability distributions in Minitab, we find the one-sided p-value for $t = -5.43026$ with 39 degrees of freedom is 0.0000016. ∎

The contrast above counts each population equally and is fundamentally different from pooling the data from judges A-F. A way that is more philosophically akin to pooling the data from Judges A-F is to use weights (c_i's) for the contrast that reflect the sample sizes. That is we would want

$$c_A \propto 5,\ c_B \propto 6,\ c_C \propto 8,\ c_D \propto 2,\ c_E \propto 7,\ c_F \propto 9$$

and thus the weights would be

$$c_A = 5/37,\ c_B = 6/37,\ c_C = 8/37,\ c_D = 2/37,\ c_E = 7/37,\ c_F = 9/37$$

In this case, the results will not change much. But we will discuss in class the different philosophies behind each of these choices of weights.

Exercises for Lecture 13

1. –
2. –