

## Practice Final - Cumulative over all material

1. Go over practice exams 1-3.
2. Give the definition of
  - (a) tangent space
  - (b) Taylor polynomial
  - (c) Quadratic form
  - (d) signature
  - (e) positive definite
  - (f) extremum
  - (g) critical point
  - (h) curvature
  - (i) torsion
3.
  - (a) Show that the set of points in  $\mathbb{R}^2$  that satisfy  $x^4 + y^3 = 15$  is a smooth manifold.
  - (b) Give the equation of the tangent space at  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
4.
  - (a) Find the Taylor polynomial of degree 2 for  $\log(1 + x)$  at  $x = 0$ .
  - (b) Find the Taylor polynomial of degree 2 for  $\log(1 + x + xy + 2y)$  at  $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
5. Compute the signature of the quadratic form  $xy$  on  $\mathbb{R}^2$ .
6. Classify the critical points of  $f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = x^4 + y^4 - 4xy + 2$ .
7. At what point does  $y = e^x$  have maximum curvature? What happens to the curvature as  $|x| \rightarrow \infty$ ?