

## Practice Exam 1

1. Give the definition of

- (a) A function
- (b) One-to-one
- (c) Onto
- (d) A convergent sequence
- (e) A linear transformation
- (f) The inverse of a matrix  $A$
- (g) The length of a matrix
- (h) A subspace of  $\mathbb{R}^n$
- (i) A vector field

2. Negate the statements

- (a) For every  $\epsilon > 0$  there exists an  $N$  so that whenever  $n \geq N$ ,  $|a_n - a| < \epsilon$ .
- (b) For every  $\epsilon > 0$  and every  $x \in \mathbb{R}$ , there exists a  $\delta > 0$  so that for all  $n$  and for all  $y$  so that  $|x - y| < \delta$ ,  $|f_n(x) - f_n(y)| < \epsilon$ .

3. Prove that the sequence  $(-2)^n$  has no limit.

4. Prove that linear transformations are continuous.

5. Are the following linear transformations? Why or why not?

(a)  $T \left( \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right) = \begin{bmatrix} v_1 v_2 \\ v_1 + v_2 + v_3 \end{bmatrix}$

(b)  $T \left( \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \right) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

6. Find the area of the parallelogram with two sides extending from  $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$  to  $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$  and from

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{ to } \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

7. Show that matrix multiplication is not commutative.

8. Give an example of a nontrivial subspace of  $\mathbb{R}^3$ .

9. What is  $4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \dots$ .

10. Show that if  $f : [a, b] \rightarrow [a, b]$  is continuous, then there exists a  $c \in [a, b]$  so that  $f(c) = c$ .