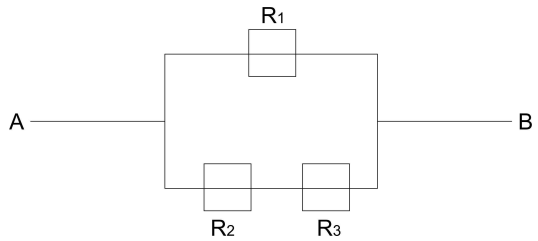
Practice Final

Name:_____

(Many problems and datasets are modeled after examples in Rice's Mathematical Statistics and Data Analysis.)

1: Polygraph tests are routinely administered to employees who hold sensitive positions. Assume that according to tests about the reliability of polygraphs, the probability a polygraph detects a person is lying when the person really is lying is about 0.88. The probability a polygraph indicates a person is truthful when the person really is truthful is about 0.86. Suppose the vast majority of employees have no reason to lie on routine polygraphs; that is, suppose the probability a randomly chosen person lies on a routine polygraph is 0.01. What is the probability that a person with a bad routine polygraph result (meaning the polygraph was interpreted as indicating the person was lying) was actually telling the truth?

2: Suppose a circuit has three relays as given in the diagram below. Each relay works or fails to work independently of the other relays. Current flows through the circuit if it can pass through all the circuits in any path from A to B. If each circuit works with probability p, what is the probability current will flow from A to B?



3: An urn contains 3 red balls, 4 blue balls, and 2 green balls. Let X denote the number of red balls in a collection of 4 balls drawn from this urn without replacement. Give the probability distribution on X, its mean, and its variance.

4: Consider the problem above. Again, let X denote the number of red balls in a collection of 4 balls drawn from the urn without replacement. Let Y denote the number of green balls drawn from this urn without replacement. Give the joint distribution of X and Y and the correlation between X and Y.

5: A Gamma distribution with scale 1 and shape parameter α has density function $f(x) = \dfrac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}$. Use integration by parts to show that the expected value of such a Gamma distribution is α.

6: Give an example of random variables X and Y that are dependent but uncorrelated.

7: What is the 95^{th} percentile of the standard normal distribution? What is the 95^{th} percentile of a normal distribution with mean 100 and standard deviation 10? What is the 95^{th} percentile of the uniform distribution on the interval $[4, 7]$? What is the 95^{th} percentile of the exponential distribution with rate 2?

8: Why is the real margin of error in an opinion poll greater than the one provided by mathematical theory?

9: What sort of considerations do you make before accepting a researcher's claim, for instance when a researcher reports that drinking 3 glasses of skim milk a day leads to greater weight loss than drinking no milk does?

10: Suppose that you want to give a 95% confidence interval for the mean of a distribution with a margin of error of ± 1. You know the standard deviation of the distribution is about 10. What sample size do you need to take to accomplish this goal?

11: How well does the normal approximation with continuity correction to the Binomial work for estimating $P(3 < X < 7)$ when X is a Binomial random variable with 10 trials and probability 0.3 of success?

12: A psychological experiment was done to investigate the effect of anxiety on a person's desire to wait alone. A group of 30 subjects was divided into two, uneven, subgroups. The first group is called the high anxiety group because they were told they would be subjected to painful electric shocks. The second group is called the low anxiety group because they were told they would be subjected to mild, painless shocks. Each individual in each group was told that the experiment would begin in 10 minutes and given the option of waiting alone or with the other subjects. The research question is whether there is a difference between the two groups in their desire to be alone. The data are as follows:

	wait together	wait alone
high anxiety	12	5
low anxiety	4	9

Name three appropriate statistical tests for this questions. Which of these tests are mathematically equivalent? Which test is most appropriate in your opinion and why? Carry out the most appropriate test.

13: Cigarette smoking can cause blood platelets to aggregate and form clots. The following data are the before and after smoking a cigarette percentages of platelets that aggregate for 11 subjects. Conduct a sign test to determine if cigarette smoking causes an increase in aggregation.

before	25	25	27	44	30	67	53	53	52	60	28
after	27	29	37	56	46	82	57	80	61	59	43
difference (after - before)	2	4	10	12	16	15	4	27	9	-1	15

14: In the above experiment with 11 subjects, what is the power of the Binomial sign test to detect a difference in the before and after percentages of platelet aggregation if the true percent of times that there is a positive difference is 80%?

15: One teacher keeps a record of the average of students' regular test averages versus their score on a common departmental final exam. For a group of 25 students, she finds the following summary data: the mean of the students' regular exams is 75 with a standard deviation of 8. The mean of the students' final exams is 60 with a standard deviation of 15. The correlation between the regular exam averages and final exam scores is 0.80. Using least squares regression give the best estimate of the final exam score for a student with a regular exam average of 85. What is a confidence interval for that student's final exam score?

Review past practice exams, past exams, and past quizzes for additional topics and problems.