Practice Exam 2

Name:_____

1: Find the mean and variance of a random variable X with probability density function $f(x) = 2e^{-2x}$ for $x > 0$. Show all of your work.

2: Let X be a Binomial random variable with 16 trials and probability 1/2 of success.

a) Find $\Pr(4 < X < 12)$ exactly.

b) Find $\Pr(4 < X < 12)$ using the normal approximation to the Binomial.

c) What is the lower bound on $\Pr(4 < X < 12)$ given by Chebyshev's inequality?

3: An American roulette wheel consists of 18 red slots, 18 black slots, and 2 green slots (numbered 0 and 00). The red and black slots are numbered 1-36 as depicted in the adjacent diagram. A bet of \$1 on "red" in American roulette yields 1 with probability 18/38 and -1 with probability 20/38. Let X denote this random yield. A split bet of \$1 between numbers, say the black number 20 and the red number 23, yields 17 with probability 2/38 and -1 with probability 36/38. Let Y denote this random yield.

a) Find the joint distribution of X and Y.

b) Find $E(X + Y)$.

c) Are X and Y independent?

d) Find the correlation between X and Y.

4: A box consists of 10 items. Some may be defective. A test for whether an item is defective or not does not harm the item. There are two inspectors who work at different times. Each inspector takes one item and tests to see whether it is defective or not. When the inspector is done, he returns the item to the box. A box is rejected if either inspector finds a defective item.

We are happy with boxes that contain 20% or fewer defective items. What is the Type I error of the test described above? What is the power of the test to detect a box that contains 50% defective items?

5: Consider the same situation as above but assume that a test for whether an item is defective or not destroys the item. Again, we are happy with boxes that contain 20% or fewer defective items. What is the Type I error of the test described above? What is the power of the test to detect a box that contains 50% defective items?