Practice Exam 1

1. State the Intermediate Value Theorem

2. Let

$$P(t) = \frac{100e^t}{e^t + 100}$$

describe the size of a population at time t. Find the inverse function and explain its meaning. When will the population reach size 10?

3. Find the domain and the range of

$$f(x) = \frac{x}{\sqrt{x^2 - 1}}.$$

Sketch a graph of the function.

4. Prove:

$$\lim_{x \to 1} (30x - 2) = 28$$

5. Either find the following limit or determine that the limit does not exist:

$$\lim_{x \to \infty} \left(\sqrt{x^2 + 2} + \sqrt{x^2 + 3}\right)$$

6. Either find the following limit or determine that the limit does not exist:

$$\lim_{x \to 0} \frac{x}{|x|}$$

7. Either find the following limit or determine that the limit does not exist:

$$\lim_{x \to 0} \left(\frac{1}{x\sqrt[3]{1 + x}} - \frac{1}{x} \right)$$

8. Either find the following limit or determine that the limit does not exist:

$$\lim_{x \to \infty} \frac{2 - x}{\sqrt{4x^2 + 13}}$$

9. Which values of x in the interval $[0, 2\pi]$ satisfy

$$2 + 2\cos(x) - \sin^2(x) = 0$$

10. Let

$$f(x) = \begin{cases} \cos(\pi x) & \text{if } x \leq 0 \\ x^2 + 2 & \text{if } x > 0 \end{cases}$$

Is there some x in $(-1, 1)$ for which $f(x) = 0$? Why or why not?