1) Which of the following is equal to

$$C(n, n-2)$$

for all values of $n \geq 2$?

a) $2!P(n, n-2)$ b) $n(n-1)$ c) $n(n-1)(n-2)$

d) $\frac{n(n-1)}{2}$ e) $\frac{n(n-1)}{(n-2)!2!}$ f) none of the above

$$C(n,n-2) \overset{\text{BY}\bigstar}{=} \frac{n!}{(n-2)!\,(n-(n-2))!} = \frac{n!}{(n-2)!\,2!} = \frac{n(n-1)}{2!} = \frac{n(n-1)}{2}$$

$$\bigstar \quad C(n,r) = \frac{n!}{(n-r)!\,r!}$$

2) A small island nation has license plates that consist of 2 letters (A - Z allowed) followed by 3 numbers (0 - 9 allowed). How many plates are possible that <u>do not repeat the letter</u>?

**Examples: AF 573, or YD 820, or NM 225.**

**<u>Nonexamples</u>: AA 234 or FF 677.**

a) 51,000 b) 2625 c) 676,00

d) 650 e) 650,000 f) none of the above

10 CHOICES

← LICENSE PLATE TO BE FILLED IN

26 CHOICES

25 CHOICES

$$26 \cdot 25 \cdot 10 \cdot 10 \cdot 10 = 650{,}000$$

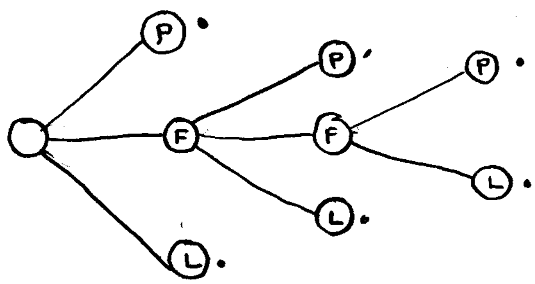
3) A car lot contains <u>three</u> Porsches, <u>two</u> Ferraris, and <u>one</u> Lotus S7. One car after another is removed from the lot (and kept off the lot). A record is kept of the make of car (Porsche, Ferrari, or Lotus) in the order of removal. The process is terminated whenever a Porsche <u>or</u> a Lotus appears on the list. What is the size of the corresponding sample space?

**Example: One outcome would be FP, indicating that first a Ferrari was chosen, then a Porsche.**

a) 6 b) 9 c) 5

d) 4 e) 7 f) none of the above

Draw a tree:




6 dots

Answer: 6.

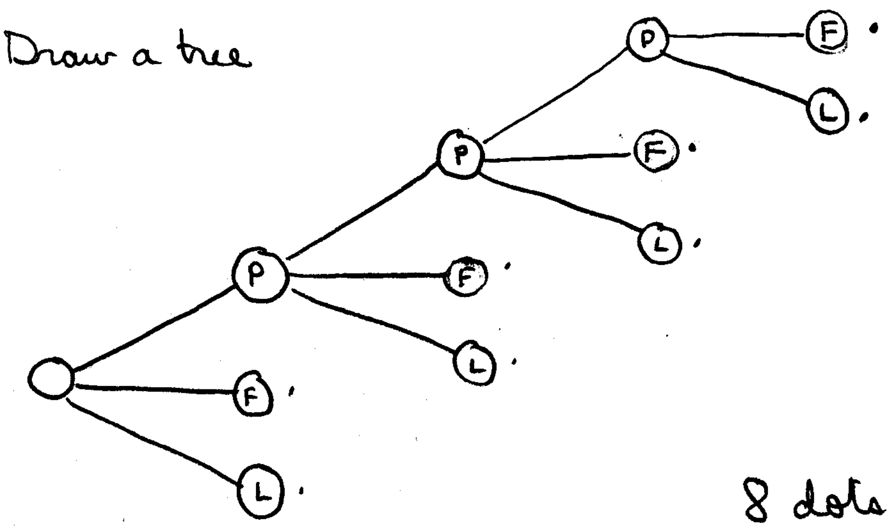
3) A car lot contains <u>three</u> Porsches, <u>two</u> Ferraris, and <u>one</u> Lotus S7. One car after another is removed from the lot (and kept off the lot). A record is kept of the make of car (Porsche, Ferrari, or Lotus) in the order of removal. The process is terminated whenever a Ferrari <u>or</u> a Lotus appears on the list. What is the size of the corresponding sample space?

**Example: One outcome would be PF, indicating that first a Porsche was chosen, then a Ferrari.**

a) 8 b) 5 c) 4

d) 6 e) 7 f) none of the above

Draw a tree




8 dots

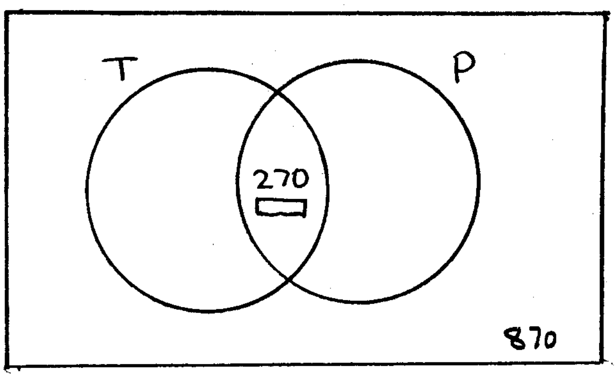
Answer: 8

4) A group of 1100 people apply to be contestants on the latest reality TV show. Of these 1100 people, 710 have tattoos and 430 have body piercing. 230 of the people don't have either (no tattoos, no piercing). How many have tattoos but no piercing?

VERSION II PIERCING BUT NO TATOOS?

a) 270 b) 160 c) 230

d) 870 e) 440 f) none of the above




$n(T \cup P) = 1100 - 230 = 870$

$n(T \cup P) = n(T) + n(P) - n(T \cap P)$
$870 = 710 + 430 - n(T \cap P)$
$\Rightarrow 870 = 1140 - n(T \cap P)$
$\Rightarrow n(T \cap P) = \boxed{270}$

$\Rightarrow n(\underbrace{T \cap P^c}_{\text{SET OF PEOPLE WITH TATOOS BUT NO PIERCING}}) = 710 - 270 = \underline{\underline{440}}$

ANSWER: VERSION I

T = set of people with tattoos
P = set of people with body piercing

ANSWER VERSION II

NOTE: $n(P \cap T^c) = n(P) - n(P \cap T) = 430 - 270 = \underline{\underline{160}}$

# with piercing but no tatoos

5) You wish to visit 4 of the 6 cities Amsterdam, Berlin, Copenhagen, Dusseldorf, Edinburgh, and Frankfort. You make out a list of the four cities that you will visit in the order that they will be visited. In how many ways can this be done?

**Example: One way is AFCD.**

a) 36 b) 300 c) 360

d) 15 e) 720 f) none of the above

_ _ _ _ ← ITINERARY

6 CHOICES 5 CHOICES 4 CHOICES 3 CHOICES

$$6 \cdot 5 \cdot 4 \cdot 3 = P(6,4) = 360$$

6) You are going on a trip with Alan, Betty, Carmen, Dorf, Edna, Frieda, and Gabi. You are to select two of these people to ride in your car. How many choices do you have if Alan and Betty insist on riding in the same car (if you don't pick both Alan and Betty, they'll ride with someone else)?

**Example: One choice would be Dorf and Carmen. Another would be Alan and Betty. Order of choice doesn't matter. Alan and Frieda would not be a viable selection.**

a) 42 b) 10 c) 14

d) 11 e) 21 f) none of the above

One choice is pick A+B.

All other choices don't have A and don't have B, that is, all other choices consist of C, D, E, F, G.

There are $\binom{5}{2}$ ways to pick 2 of the 5 people C, D, E, F, G.

Answer:

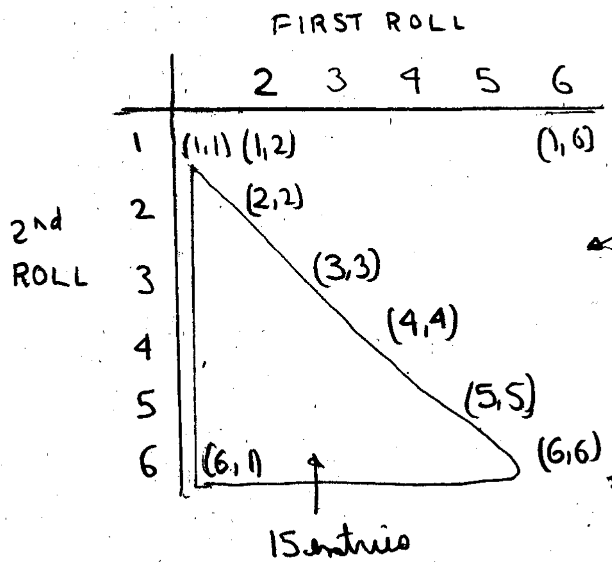
$$\underbrace{1}_{\text{PICK A+B}} + \underbrace{\binom{5}{2}}_{\text{PICK TWO OF CDEFG}} = 1 + 10 = 11$$

7) A fair die is rolled two times. What is the probability that the second roll is HIGHER than the first?

**Example: Roll a 3 then a 5. The second roll is higher than the first. Rolling a 4 and then a 4 would not "count" since 4 is not higher than 4.**

a) 31/36 b) 13/18 c) 5/6

d) 2/3 e) 5/12 f) none of the above




36 entries on chart showing outcomes

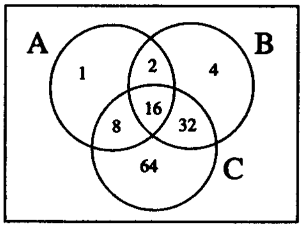
6 entries have the 2 rolls equal

$\Rightarrow$ $36-6=30$ outcomes have one roll higher than the other

$\Rightarrow$ $\frac{30}{2}=15$ outcomes have the 2nd roll higher than the 1st.

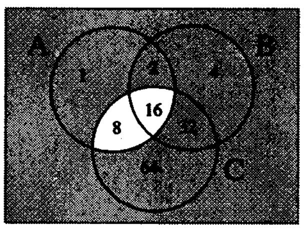
Answer: $\frac{15}{36} = \frac{5}{12}$

8) Let A, B, C be subsets of a universal set $\Omega$ where $n(\Omega) = 214$. Shown below is a Venn diagram for the sets A, B, C (which has been labelled with the number of elements in its various subsets). How many elements are in the set $(A \cap C)' \cup (B \cap C)$?

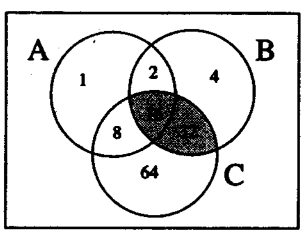



a) 206 b) 186 c) 212

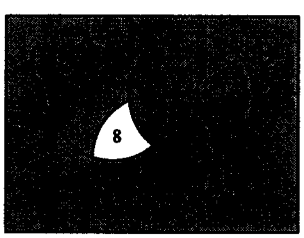
d) 198 e) 24 f) none of the above




← $(A \cap C)'$




← $(B \cap C)$



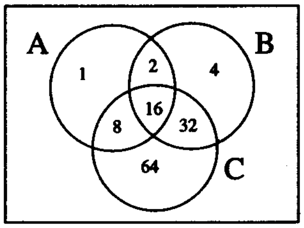

214

$(A \cap C)' \cup (B \cap C)$

This is the superposition of the 2 diagrams given above.

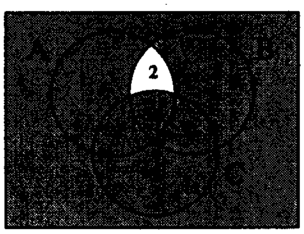
Answer: $214 - 8 = 206$

8) Let A, B, C be subsets of a universal set $\Omega$ where $n(\Omega) = 214$. Shown below is a Venn diagram for the sets A, B, C (which has been labelled with the number of elements in its various subsets). How many elements are in the set $(A \cap B)' \cup (B \cap C)$?




a) 212 b) 186 c) 206

d) 198 e) 18 f) none of the above




← Same reasoning as on the other version.

Answer: $214 - 2 = \underline{\underline{212}}$

9) How many 9 letter "words" can be formed using 3H's, 2T's, and 4O's?

**Examples:** HTHHOOTOO or OTHOHOTHO.

a) 630 b) 1260 c) 144

d) 84 e) 512 f) none of the above

$$\frac{9!}{3!\,2!\,4!} = \frac{9\cdot 8\cdot 7\cdot 6\cdot 5}{6\cdot 2} = 9\cdot 4\cdot 7\cdot 5 = 1260$$

10) There are ten (10) people working under you. You have to select one person to be your assistant and two people to be transferred to another department. In how many ways can this be done?

**Examples: One way is: Promote person 1 and transfer 5 and 6. Another way is: Promote 3 and transfer 1 and 8. Note that transferring 1 and 8 is the same as transferring 8 and 1.**

a) 240 b) 260 c) 720

d) 120 e) 360 f) none of the above

There are 10 ways to select one person to be your assistant. Having done that, there are 9 people left over. There are $\binom{9}{2}$ ways to select 2 of these people to be transferred.

Answer:

$$10 \cdot \binom{9}{2} = 10 \cdot \frac{9 \cdot 8}{2} = 360.$$

11) Suppose $\Omega$ is a universal set with $n(\Omega) = 200$, and suppose $A$, $B$, and $C$ are subsets of $\Omega$ with:

$n(A) = 55$ $\quad$ $n(B) = 65$ $\quad$ $n(C) = 75$

$n(A \cap B) = n(B \cap C) = n(A \cap C)$

$n(A \cap B \cap C) = 4$

$n(A \cup B \cup C) = 157$

What is $n(A \cap C)$?

a) 12 $\quad$ b) 19 $\quad$ c) 184

d) 14 $\quad$ e) 16 $\quad$ f) none of the above

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$

Set $x = n(A \cap B) = n(B \cap C) = n(A \cap C)$. Then the above equation becomes

$157 = 55 + 65 + 75 - x - x - x + 4$

$\Rightarrow 157 = 199 - 3x \Rightarrow 42 = 3x \Rightarrow x = \underline{\underline{14}}$.

11) Suppose $\Omega$ is a universal set with $n(\Omega) = 200$, and suppose $A$, $B$, and $C$ are subsets of $\Omega$ with:

$n(A) = 55$ $\quad$ $n(B) = 65$ $\quad$ $n(C) = 75$

$n(A \cap B) = n(B \cap C) = n(A \cap C)$

$n(A \cap B \cap C) = 10$

$n(A \cup B \cup C) = 169$

What is $n(A \cap C)$?

a) 14 $\quad$ b) 19 $\quad$ c) 184

d) 12 $\quad$ e) 16 $\quad$ f) none of the above

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(B \cap C) - n(A \cap B) + n(A \cap B \cap C)$

Set $x = n(A \cap B) = n(B \cap C) = n(A \cap C)$. Then the above equation becomes

$169 = 55 + 65 + 75 - x - x - x + 10$

$\Rightarrow 169 = 205 - 3x \Rightarrow 36 = 3x \Rightarrow x = \underline{\underline{12}}$

12) An incomplete deck of cards consists of the 4 aces, the four two's, and the four three's. You draw 3 cards at random from this incomplete deck, without replacement. What is the probability that you drew at least 1 ace?

a) 41/55 b) 41/110 c) 69/110

d) 3/110 e) 107/110 f) none of the above

Let

E = event of at least 1 ace.

Then

E' = event of no aces and $Pr(E) = 1 - Pr(E')$

$$Pr(E') = Pr(\text{no aces}) = \frac{\binom{8}{3}}{\binom{12}{3}}$$

$\binom{8}{3}$ ← # ways to choose 3 of the 8 "non-ace" cards

$\binom{12}{3}$ ← # ways to choose 3 cards of the 12

$$\Rightarrow \quad Pr(E') = \frac{8\cdot 7\cdot 6/3!}{12\cdot 11\cdot 10/3!} = \frac{8\cdot 7\cdot 6}{12\cdot 11\cdot 10} = \frac{14}{55}$$

So

$$Pr(E) = 1 - \frac{14}{55} = \underline{\underline{\frac{41}{55}}}.$$

13) Someone ripped the labels off of your soup cans. You have 4 cans of <u>split pea</u> and 5 cans of <u>tomato</u> soup. You select 3 cans at random from the 9 cans, without replacement. What is the probability that you got 2 cans of split pea and 1 can of tomato?

a) 5/7 b) 4/9 c) 5/21

d) 10/21 e) 5/14 f) none of the above

# ways to select 2 of 4 cans of split pea

# ways to select 1 of the 5 tomato soup cans

$$\frac{\binom{4}{2}\binom{5}{1}}{\binom{9}{3}}$$

# ways to select 3 of the 9 cans.

$$= \frac{6 \cdot 5}{9 \cdot 8 \cdot 7 / 3!} = \frac{30}{3 \cdot 4 \cdot 7} = \frac{5}{14}$$

13) Someone ripped the labels off of your soup cans. You have 4 cans of <u>split pea</u> and 5 cans of <u>tomato</u> soup. You select 3 cans at random from the 9 cans. What is the probability that you got 1 can of split pea and 2 cans of tomato?

a) 5/7 b) 4/9 c) 5/21

d) 5/14 e) 10/21 f) none of the above

# ways to select one of 4 split pea

$$\frac{\binom{4}{1}\binom{5}{2}}{\binom{9}{3}}$$

# ways to select 2 of the 5 tomato soup cans

# ways to select 3 of the 9 cans.

$$= \frac{4 \cdot 10}{\binom{9}{3}} = \frac{40}{9 \cdot 8 \cdot 7 / 3!} = \frac{40}{3 \cdot 4 \cdot 7} = \frac{10}{21}$$

14) A vase contains **four red flowers** and **five yellow flowers**. You select three of the flowerss at random, one after the other, without replacement. What is the probability that you got a red, then a yellow, then a yellow?

a) 25/126 b) 10/21 c) 5/42

d) 10/63 e) 5/13 f) none of the above

# ways to select one red flower

# ways to select one yellow flower

There are 3 yellows left

$$\frac{4 \cdot 5 \cdot 3}{9 \cdot 8 \cdot 7}$$

# ways to select 3 of the 9 flowers, keeping track of order. } P(9,3)

$$= \frac{5}{3 \cdot 2 \cdot 7} = \frac{5}{42}$$

15) There are 5 teams of 4 people each. How many ways can you select 3 of the 20 people so all three people are from different teams?

**Example: Select the $4^{th}$ person on team 1, the $2^{nd}$ person on team 2, and the $2^{nd}$ person on team 5 - one person has been selected from each of three different teams.**

**Hint: Select 3 teams first. (How many ways can the 3 teams be selected?) Next select one person from each of those teams.**

a) 38,400 b) 640 c) 192

d) 64 e) 1920 f) none of the above

\# ways to select one person from one team

$$\binom{5}{3} \cdot 4 \cdot 4 \cdot 4$$

select one person from the remaining team.

select one person from another team

\# ways to select 3 of the 5 teams