FINITE MATHEMATICS, M118

SAMPLE TESTS AND STUDY HINTS

2003-2004

Introduction

About M118

Homework, quizzes, and exams are usually required of students enrolled in M118. **Dates and times of departmental exams are found in the Schedule of Classes**. You **MUST** take the departmental exams at the scheduled times. Most instructors use the following as a guide in planning exams for the semester. Use it to plan your study sessions.

Exam	Material Covered	Date
1	Chapters 1 and 2	See Instructor
Departmental Midterm	Chapters 1, 2 and 3 and Sections 4.1 and 4.2	See Schedule of Classes
3	Chapters 5, 6 and 7	See Instructor
Departmental Final	Chapters 1, 2, 3, 4, 5, 6, 7 and Sections 8.1-8.3	See Schedule of Classes

Your Background

Prerequisites: fractions, algebra, and a willingness to work.

Fractions.

You should be able to work with fractions quickly and correctly. You must be able to reduce a fraction to "lowest terms" to match the multiple choice answers on departmental exams. If you are uncomfortable with the following examples, you'll need more math before you take M118.

$$\frac{72}{111} = \frac{24}{37}$$
$$\frac{2}{7}+\frac{4}{9} = \frac{46}{63}$$

Algebra.

You need to know enough algebra to understand how an equation is transformed step-by-step into an equivalent equation. For example,

$$\text{``}-3x+6=15\text{''} \quad \text{to} \quad \text{``}x=-3\text{''}.$$

Work.

The most important prerequisite is a willingness to think long and hard about a given problem before giving up.

Dice and Cards.

You also need to know about dice and playing cards. A **fair die** is an ordinary 6-sided die (as used in Backgammon, Monopoly, Trivial Pursuit, etc.). More precisely, it is a cube marked with symbols for numbers in the set $\{1,2,3,4,5,6\}$ having the property that each number is as likely to be on top of the die as any other number when the cube is thrown on a tabletop (an unfair die would not have this property). Separate rolls of a die (fair or not) are usually assumed to be independent (earlier rolls do not affect later ones.).

A **standard deck** is made up of 52 cards. Each card has a **suit** and a **rank**; there are 4 suits and 13 ranks. The **red** suits are **Diamonds** and **Hearts**. The **black** suits are **Spades** and **Clubs**.

Course Outline

Chapter 1.

Most of this chapter deals with **sets**. A knowledge of sets, and how to represent and combine them, is important throughout the course, especially in the study of probability. You need to learn the **vocabulary** and **notation** of **set theory**. Among the other important concepts in this chapter are **tree diagrams**, **partitions**, and the **multiplication principle**.

Chapter 2.

This chapter deals mainly with counting. The techniques for counting are many and varied. In each problem, you will have to determine which techniques to use, or even devise techniques of your own, and then apply the techniques correctly. There is no recipe which covers all cases. Although memorizing answers to certain problems is not very helpful, remembering how you solved a certain problem may be useful. You must understand how to get from story problems to formulas, and then how to carry out the calculations needed to reach a final answer. More time will be needed for study of Chapter 2 than for Chapter 1.

- **EXAMINATION 1** covers Chapters 1 and 2.

Chapter 3.

This chapter contains the basic ideas of **probability**, and involves new concepts and notation. One of the main ideas is that of **conditional probability**. A fundamental tool is the **tree diagram**, with which many of the exercises in the chapter can be solved. This chapter also considers a special kind of sequential experiment, called a **Bernoulli trial**.

Chapter 4.

The M118 course covers sections 4.1 and 4.2. The chapter deals with the important idea of a **random variable** and the associated **density function**. The **expected value** (or average) if a random variable is widely used in applications. The chapter also covers the **standard deviation** of a random variable.

- **MIDTERM EXAMINATION** covers Chapters 1, 2, and 3 and Sections 4.1 and 4.2. This is a departmental exam!

LAST DROP DATE - The last date for an AUTOMATIC "W" for all IU classes is shortly after the midterm exam. You won't be able to drop any courses after this time without a Dean's permission. If you are failing or otherwise unhappy with your grade, you should consider withdrawing from the course by this date. The Math Department is quite strict about giving grades of "Incomplete", and you definitely will not qualify unless you complete all but the very end of the course with a passing grade, and have a documented reason why it is impossible for you to complete the course.

Chapter 5.

For most students, the beginning of this chapter is a review from previous courses. If you haven't worked with **graphs of lines** before, it would be a good idea to look ahead at the first section of the chapter before it is discussed in class. Your instructor will probably cover the material very quickly. The rest of the chapter is devoted to solving **systems of linear equations**, first with two variables and then with any number of variables. **It is important to learn the details of the general method**, even though you may already know a simpler method which works in special cases. The general method is used again in Chapters 6 and 8 to find inverses of matrices.

Chapter 6.

This chapter introduces the idea of a **matrix** and shows how matrices are related to systems of linear equations. Basic operations are defined. Section 6.3 covers an important application of matrices in economics.

- **EXAMINATION 3** covers Chapters 5, 6, and 7.

Chapter 7.

This chapter considers **linear programming** problems in two variables. These are problems which involve finding the maximum or minimum of a linear function. It is very important to learn how to set these problems up correctly. The problems are solved by using graphs of lines in the plane and using points on these graphs to evaluate the **objective function**.

- **FINAL EXAMINATION** covers the entire course. This is a departmental exam!

Sample Exams

Although the exams in this pamphlet are mostly multiple-choice questions, **the exams in your class will not necessarily be multiple-choice**. Some instructors use a mixture of multiple-choice and "show your work" problems. However, the departmental midterm and final will consist mostly of multiple-choice questions. The exams in this booklet were selected from those given to classes in recent years, and were written by a variety of professors.

Some Study Hints

Learning Finite Mathematics takes work. Here are some useful hints:

- **Do problems!** If you do, and understand every exercise in the textbook, the exams will be mostly straightforward and familiar to you. The exams in M118 focus entirely on problem solving. The level of difficulty of the problems on examinations will usually vary from one problem to another. Some problems will be easy, while some may be quite challenging.

- **Study the examples** in the textbook. Do not be reluctant to ask questions. If parts of the examples are unclear to you, ask your instructor or assistant. Instructors expect questions and are willing to help you to understand the examples.

- **Do easy exercises first.** The problems in each section generally get harder as you go along. Sometimes knowing how to solve an earlier problem helps in solving a later one. If you have trouble with number 15, then you probably haven't done enough problems from 1-14.

- **Make up easy examples similar to difficult problems.** This helps you understand what's really going on in a problem. How many whole numbers are there between 5 and 173 (Including 5 and 173)? It's not 168. You could write them all down and count them, but it's easier to consider a simpler example — how many whole numbers are there between 5 and 7 (including 5 and 7)? It's easy to see that the answer is 3 (not 2): the numbers 5, 6, and 7. We get one more than the difference by counting both 5 and 7. The same thing will happen in the big example, so we get 169 numbers between 5 and 173. Using small numbers instead of big ones is frequently very helpful.

- **Use the answers in the back of the textbook only as a check on your work.** It is useful to know whether you have the correct answer to a problem. For that reason, the answers to the odd-numbered exercises are given in the text. You should not use these answers to determine how to solve a problem. Solve the problem first, and then check whether your answer is correct.

- **Study for understanding.** The major goal of this course is for you to learn to solve problems. It is not enough to solve a problem by finding a similar example worked in the text or in your lecture notes and plugging in different numbers. After all, you won't have your text and notes available during exams! You should study the text and your notes until you think you understand the new ideas, and then test your understanding by working the exercises.

- **Read actively.** Read the material on each subject in the text before the material is covered in class so you will be prepared to understand what your instructor says in each lecture. This makes it easier to take notes and to focus your attention on the more difficult material. Reread a portion of the book after some later material has had a chance to "sink in". If you don't understand, mark it and be sure to keep coming back to try again until you do understand it. Read with your pencil in hand; work out the exercises in the margin as you go.

- **Study the vocabulary** of mathematics. Be able to explain each of the "Important Terms and Concepts" listed in the last section of each chapter. Know how to pronounce special symbols out loud (for example, "5!" is pronounced "five factorial"). Each term is discussed in the text, and defined in the glossary near the end of the textbook.

- **Take advantage of help sessions and the office hours of the instructors and assistants.** At the beginning of the semester you will be given information about the times during which help is available; make careful note of the schedule, and use these services frequently. It is silly to spend hours puzzled when a visit to a help session or an office hour can clear things up. Don't be afraid to ask questions – people are there to help. Being prepared with specific questions or problems can make your visit more productive.

- **Form study groups.** Talking about mathematics helps one learn mathematics, so many people find study groups (say of 3 or 4 people) helpful. Typically students try the problems individually and then talk about them in the group. On the other hand, you will take exams and quizzes alone, so make sure your group works efficiently and is not an excuse for gabbling or for people avoiding doing the exercises themselves.

Taking Examinations

Be prepared.

The most important part of taking an exam is to be sure you are prepared. In addition, here are some things you can do to improve your performance.

- Look over the examination, make a general time allocation, and stick to it. If you have a 15-question examination in a 50-minute class, you have an average of slightly less than 4 minutes per question. You probably cannot afford to spend 15 minutes on one question. You should plan to complete about half the questions in 25 minutes, which is half the time available.

- Read through the exam and work the problems that are most familiar to you first.

- Answer all multiple-choice and true/false questions. On multiple-choice tests, guessing is a better strategy than leaving an answer blank.

- Be sure to show work if you are asked to do so.

- Try to leave enough time to review your work.

- **Do your own work.** It is department policy to enforce general university policies on academic honesty.

Sample Exams and Quizzes

To get the most benefit from this booklet, it is recommended that you take each of the included exams and quizzes under realistic test conditions. In other words,

- Try to get through the entire exam in the specified time.

- The first time you go through each exam, you should work alone.

- Find out if calculators will be allowed on the exam. If they won't, you shouldn't use one on the practice exam, either.
- After the time is up (and only then), use the answers in the back of this booklet to check your work and help clear up any points of confusion.

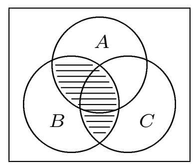
Exam 1 – Version 1

This is a 75 minute test.

1. Consider the universal set $U = \{a, b, c, d, 1, 2, 3\}$ along with subsets $A = \{a, b\}$, $B = \{c, 1\}$ and $C = \{a, 1, 2, 3\}$. Find the set $(A \cup (B \cap C))'$.

   (A) $\{a, 1\}$ (B) $\{c, d, 2\}$ (C) $\{c, d, 2, 3\}$
   (D) $\{b, c, d, 2\}$ (E) $\{b, c, d, 2, 3\}$ (F) none of these

2. Identify the shaded set in the following Venn diagram.



   (A) $(A \cap B) \cap (B \cup C)$ (B) $(A \cap B) \cup (B \cap C)$ (C) $(A \cap C) \cup (B \cap C)$
   (D) $(A' \cap B) \cup C$ (E) $A \cap B' \cap C$ (F) none of these

3. Determine which of the following forms a partition of $U = \{a, b, c, d, 1, 2, 3\}$

   (A) $\{A, B, C\}$, where $A = \{a, 1, 3\}$, $B = \{b, c, 2\}$, $C = \{d, 2\}$
   (B) $\{A, B, C\}$, where $A = \{a, 1, 3\}$, $B = \{b, c, 2\}$, $C = \{d, 3\}$
   (C) $\{A, B, C\}$, where $A = \{a, 3\}$, $B = \{b, c\}$, $C = \{d, 1, 2\}$
   (D) $\{A, B, C\}$, where $A = \{a, b\}$, $B = \{c, d\}$, $C = \{1, 2\}$
   (E) $\{A, B, C\}$, where $A = \{a, b, c, d\}$, $B = \{1, 2, 3\}$, $C = \varnothing$
   (F) none of these

4. Let $A = \{x, y, z\}$, $B = \{1, 2\}$, and the universal set $U = A \times B$. Find C', where $C = \{(x, 1), (x, 2), (z, 2)\}$.

   (A) $\{(z, 1), (y, 2), (z, 2)\}$ (B) $\{(z, 1), (y, 2)\}$ (C) $\{(z, 2), (y, 2), (y, 1)\}$
   (D) $\{(y, 1), (y, 2), (z, 2)\}$ (E) $\{(z, 1), (y, 2), (y, 1)\}$ (F) none of these

5. One hundred students are surveyed. Fifty say they want a room in a dorm, 55 want a meal plan at a dorm, and 35 want both. Determine how many students want neither a room nor a meal plan?

   (A) 10 (B) 15 (C) 20 (D) 25 (E) 30 (F) none of these

6. An experiment consists of flipping a coin at most four times and noting each time whether the result is a head or tail. The experiment is stopped if either two consecutive heads or two consecutive tails occur. Determine the total number of outcomes for this experiment. (Hint: Draw a tree diagram.)

   (A) 6 (B) 8 (C) 10 (D) 14 (E) 16 (F) none of these

7. Find $P(6, 2)$.

   (A) 20 (B) 30 (C) 40 (D) 60 (E) 120 (F) none of these

8. Find $C(10, 2)$.

   (A) 20 (B) 40 (C) 45 (D) 50 (E) 110 (F) none of these

9. Find the number of ways in which a club with 8 members can select a president and treasurer.

   (A) 17 (B) 18 (C) 56 (D) 72 (E) 81 (F) none of these

10. A cellular phone company offers 2 types of service (digital and analog), along with 3 calling plans for each type of service, and 3 styles of phones available with each type of service, and regardless of the plan. A complete package consists of a calling plan, a type of service, and a phone. Find the total number of complete packages.

    (A) 9 (B) 12 (C) 18 (D) 24 (E) 28 (F) none of these

11. Suppose an experiment has a sample space of outcomes $S = \{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4, \mathcal{O}_5\}$ with associated weights (probabilities) $w_1 = .20$, $w_2 = .15$, $w_3 = .30$, $w_4 = .25$, and $w_5 = .10$. If $E_1 = \{\mathcal{O}_1, \mathcal{O}_2\}$, $E_2 = \{\mathcal{O}_2, \mathcal{O}_4, \mathcal{O}_5\}$, find $Pr[E_1' \cap E_2]$.

    (A) .25 (B) .30 (C) .35 (D) .45 (E) .55 (F) none of these

12. A judiciary committee of 5 is to be selected from a group of 5 Democrats and 6 Republicans, in such a way that there are at least two members from each party on the subcommittee. Determine the number of ways that this subcommittee can be chosen.

    (A) 320 (B) 330 (C) 340 (D) 350 (E) 360 (F) none of these

13. There are 21 cans of cola in a cooler. Seven of them are diet cola, and the rest are regular cola. If two cans are pulled from the cooler at random, find the probability that both are diet.

    (A) $\frac{1}{10}$ (B) $\frac{3}{10}$ (C) $\frac{3}{20}$ (D) $\frac{4}{21}$ (E) $\frac{5}{17}$ (F) none of these

14. Suppose that a student walking through the Sample Gates is asked to take an Ad Sheet with probability .7, is asked to answer a survey with probability .2, and is asked to do both with probability .05. Determine the probability that a student will be asked to do neither.

    (A) .05 (B) .10 (C) .15 (D) .20 (E) .25 (F) none of these

15. A "hand" of five cards are drawn at random from standard deck of 52. Determine the probability that the hand consists of three aces and two kings.

(A) $\frac{C(4,3)\times C(4,2)}{C(52,5)}$ (B) $\frac{C(4,3)+C(4,2)}{C(52,5)}$ (C) $\frac{C(4,3)+C(48,2)}{C(52,5)}$
(D) $\frac{C(4,3)\times C(48,2)}{C(52,5)}$ (E) $\frac{C(5,3)\times C(5,2)}{C(52,5)}$ (F) none of these

16. An experiment with equally likely outcomes has a sample space S and an event E, with $Pr[E] = .25$ and $n(E) = 10$. Find $n(S)$.

(A) 40 (B) 50 (C) 60 (D) 70 (E) 80 (F) none of these

17. How many subsets with exactly 3 elements are there for a set of 8 elements?

(A) 27 (B) 42 (C) 56 (D) 64 (E) 110 (F) none of these

18. Sports fans in the state were polled and the following data obtained.

41 percent followed Indiana University sports
40 percent followed Purdue University sports
48 percent followed Notre Dame University sports
20 percent followed both Indiana and Notre Dame
25 percent followed both Purdue and Notre Dame
10 percent followed both Indiana and Purdue
5 percent followed all three.

Find the percentage of those polled who follow Indiana but neither Purdue nor Notre Dame.

(A) 15 percent (B) 16 percent (C) 17 percent
(D) 18 percent (E) 20 percent (F) none of these

19. Determine the number of five-letter code words that can be formed from the word *mommy*.

(A) 10 (B) 15 (C) 20 (D) 30 (E) 60 (F) none of these

20. The music school has 3 women and 4 men audition for solo performances. In deciding on an evening program the director must choose 4 different soloists performing *in a particular order*. If a program is selected at random, determine the probability that the program consists of two women followed by two men.

(A) $\frac{2}{35}$ (B) $\frac{3}{35}$ (C) $\frac{6}{35}$ (D) $\frac{16}{35}$ (E) $\frac{17}{35}$ (F) none of these

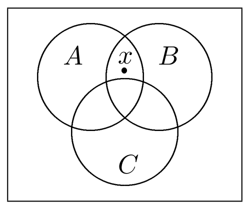
Exam 1 – Version 2

This is a 50 minute test.

1. Of a group of 100 people, 33 like earrings, 29 like tattoos, and 26 like tattoos but don't like earrings. How many like neither earrings nor tattoos?

   (A) 59 (B) 41 (C) 38 (D) 12 (E) none of the others

2. In the diagram below, which of the following is true?



   (A) $x \in (A \cap C') \cup (B \cap C)$ (B) $x \in (A \cup B) \cap (A' \cup C)$ (C) $x \in (A \cap C') \cap (B \cap C)$
   (D) $x \in (A \cap C) \cup (B' \cap C)$ (E) none of the others

3. Two 6-sided dice are rolled. What is the probability that at least one of the dice shows a 1?

   (A) $\frac{1}{3}$ (B) $\frac{1}{12}$ (C) $\frac{11}{36}$ (D) $\frac{1}{6}$ (E) none of the others

4. A saleswoman is scheduling trips to visit 3 of the following 6 cities: Cleveland, Chicago, St. Louis, Buffalo, Detroit, and Kansas City. A schedule is a list of the 3 cities in the order to be visited. How many different schedules are there which include Kansas City?

   (A) 60 (B) 120 (C) 20 (D) 24 (E) none of the others

5. Find $n(A \cap B)$, given that A and B are subsets of U with $n(U) = 52$, $n(A' \cap B') = 8$, $n(A) = 31$, and $n(B') = 10$.

   (A) 29 (B) 44 (C) 21 (D) 33 (E) none of the others

6. A Greek urn contains a red ball, a blue ball, a yellow ball, and an orange ball. A ball is drawn from the urn at random and then replaced. If one does this 4 times, what is the probability that all 4 colors were selected?

   (A) $\frac{1}{70}$ (B) $\frac{3}{32}$ (C) $\frac{2}{9}$ (D) $\frac{1}{4}$ (E) none of the others

7. A fortune teller foretells that each of 4 boys (Eric, Jason, Jeff, Kevin) will marry one of 4 girls (Jodie, Lisa, Susan, Wendy). How many ways can this be done? (No divorces or bigamy please!)

   (A) 12 (B) 6 (C) 44 (D) 24 (E) none of the others

8. How many 4-letter words can be formed by rearranging the letters in the word "seek"?

   (A) 12 (B) 2^4 (C) 6 (D) 24 (E) none of the others

9. Godzilla wants to destroy two buildings: a fast-food restaurant and either a tanning salon or a public restroom. He sees the following buildings: 2 fast-food restaurants, 3 tanning salons, and 2 public restrooms. How many ways can Godzilla fulfill his desires?

   (A) 10 (B) 21 (C) 12 (D) 7 (E) none of the others

10. I have 4 pencils, 3 ball-point pens, and a felt-tip marker in a cup on my desk. I choose a writing implement from the cup randomly. What is the probability that it is **not** a ball-point pen?

    (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{12}$ (D) $\frac{3}{8}$ (E) none of the others

11. Captain Jean-Luc Picard has 3 blue shirts and 5 red shirts. He picked two at random to take on a weekend trip to Earth. What is the probability that he took a shirt of each color?

    (A) $\frac{1}{15}$ (B) $\frac{2}{7}$ (C) $\frac{8}{15}$ (D) $\frac{15}{28}$ (E) none of the others

12. Constance Noring has 5 cassettes by the Rolling Stones and 2 cassettes by R.E.M. She picks three at random to listen to in her car. What is the probability that she takes 2 by the Rolling Stones and 1 by R.E.M.?

    (A) $\frac{2}{35}$ (B) $\frac{2}{7}$ (C) $\frac{13}{35}$ (D) $\frac{4}{7}$ (E) none of the others

13. Bob Smith watches the television talk shows on Wednesday morning. After each show he notes whether it is dull, offensive, or good. He turns off the TV after seeing 1 dull show, 1 offensive show, or if 3 shows have been watched altogether. How many outcomes are possible?

    (A) 13 (B) 23 (C) 7 (D) 9 (E) none of the others

Numbers 14–16 are True/False.

14. $n(A \cap B) + n(A \cup B) = n(A) + n(B)$ for any sets A and B.

15. $(A \cup B)' = A' \cup B'$ for any sets A and B.

16. Flip two coins and note the number of heads. The outcomes $\{0, 1, 2\}$ are equally likely.

MIDTERM EXAMS

Sample exams included in textbook, Appendix D

Exam 3 – Version 1

This is a 75 minute test.

Part 1.

Problems 1 through 13 are worth 5 points each, a total of 65 points.

In each of the problems 1 through 4, the augmented matrix shown has been obtained by a sequence of row operations. In each case, decide which of the following statements is true about the associated system of equations.

(A) The system has a unique solution.
(B) The system has no solution.
(C) The system has an infinite number of solutions with **one** arbitrary parameter.
(D) The system has an infinite number of solutions with **two** arbitrary parameters.
(E) None of the above.

1. $\left[\begin{array}{ccc|c} 1 & 0 & 5 & 10 \\ 0 & 3 & -3 & 6 \\ 0 & 1 & -1 & 2 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 1 & 0 & 6 & 4 \\ 0 & 3 & -3 & 6 \\ 0 & 1 & -1 & -2 \end{array}\right]$

3. $\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 2 \\ 0 & 1 & -1 & -4 & 0 \\ 0 & 2 & -2 & -8 & 0 \end{array}\right]$

4. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 0 & 1 & -1 & -3 & 1 \\ 0 & 0 & 1 & 1 & -2 \end{array}\right]$

5. Find the maximum value of the objective function in the following linear programming problem.

$$\text{Maximize } 3x + y, \text{ subject to } \begin{cases} x \geq 0 \\ y \geq 0 \\ 5x + 2y \leq 90 \\ 3x + 4y \leq 96 \\ y \leq 30 \end{cases}$$

(A) 96 (B) 51 (C) 54 (D) 30 (E) none of the others

For problems 6, 7, *and* 8, *the matrices* **A**, **B**, *and* **C** *are defined as*

$$\mathbf{A} = \begin{bmatrix} 4 & -2 & 1 \\ 2 & -5 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -1 & 3 \\ 3 & 1 \\ 2 & -2 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 5 & -3 \\ -1 & 2 \\ 3 & 0 \end{bmatrix}$$

6. Find the $(2,1)$ element of $\mathbf{AB}$.

   (A) -2 (B) 8 (C) 12 (D) -5 (E) none of the others

7. Find the $(1,2)$ element of $2\mathbf{B} - \mathbf{C}$.

   (A) 7 (B) 5 (C) 9 (D) 3 (E) none of the others

8. A matrix $\mathbf{D}$ satisfies $3\mathbf{B} + \mathbf{D} = 2\mathbf{C}$. Find the $(2,2)$ element of the matrix $\mathbf{D}$.

   (A) -1 (B) 1 (C) 2 (D) 3 (E) none of the others

9. Which of the following systems of equations has a unique solution?

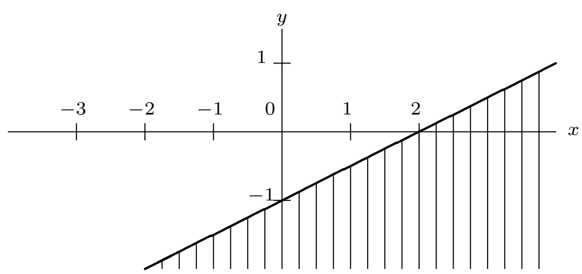
   (A) $\begin{cases} 2x - 3y = 2 \\ -4x + 6y = 4 \end{cases}$ (B) $\begin{cases} 2x - 3y = 2 \\ 4x - 6y = 4 \end{cases}$ (C) $\begin{cases} 2x - 3y = 2 \\ 4x + 6y = -4 \end{cases}$

   (D) $\begin{cases} 2x - 3y = 2 \\ -4x + 6y = -4 \end{cases}$ (E) none of the others

10. Find the y-coordinate of the intersection of the lines $3x - 4y = 11$ and $5x + 2y = 1$.

    (A) -2 (B) 1 (C) 4 (D) $\frac{1}{2}$ (E) none of the others

11. Which of the following constraints describes the shaded set shown on the coordinate system below?



    (A) $x - 2y \leq 2$ (B) $x - 2y \geq 2$ (C) $x - 2y \leq -2$
    (D) $x - 2y \geq -2$ (E) none of the others

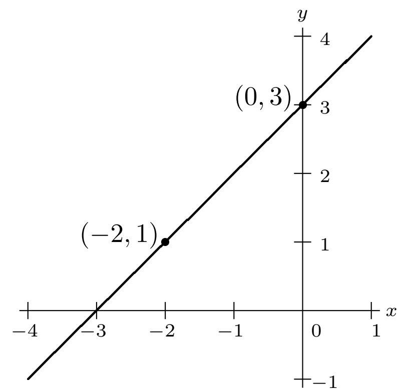
12. A set S is described by the system of inequalities

$$\begin{cases} 6x + 2y \leq 10 \\ 2x - y \geq 0 \\ x - 3y \leq 15 \end{cases}$$

    Find the y-coordinate of the corner point of S in the first quadrant.

    (A) 2 (B) 4 (C) 1 (D) 3 (E) none of the others

13. Find the equation of the line whose graph is shown below.



(A) $x - y = 3$ (B) $3x - y = -3$ (C) $x - y = -3$
(D) $x + y = 3$ (E) none of the others

Part 2.

Problem 1 is worth 15 points, problems 2 and 3 are worth 10 points each, a total of 35 points.

1. **Formulate and solve** the following problem. **Show all work**.

   Barbara's Basket's, Inc., produces two types of decorative straw baskets, Colonial and Southwest styles. One Colonial basket requires 30 pieces of yellow straw and 30 pieces of brown straw, and one Southwest basket requires 50 pieces of yellow straw and 10 pieces of brown straw. Each week the company has 4500 pieces of yellow straw and 1500 pieces of brown straw to be used to produce baskets. There is a commitment to produce at least 30 Southwest baskets. The profit is \$4.00 for each Colonial basket and \$6.00 for each Southwest basket.

   How many baskets of each type should be produced each week to maximize profit?

2. Find all solutions of the following system of equations. **Show all work**.

$$\begin{cases} 2x - 5y = 0 \\ x - 3y - z = -1 \\ -x + 2y - z = -1 \end{cases}$$

3. Find the inverse of the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}$. **Show all work**.

Exam 3 – Version 2

This is a 75 minute test.

1. Find an equation for the straight line which goes through $(2, 3)$ and is parallel to the line $9x + 3y = 1$.

   (A) $3x - y = 3$ (B) $3x + y = 9$ (C) $3x + y = 1$
   (D) $2x + y = 7$ (E) $2x - y = 1$ (F) none of these

2. Find the equation which describes a line with y-intercept -2 and slope $\frac{4}{3}$?

   (A) $3x + 4y = 6$ (B) $3x - 4y = 6$ (C) $4x + 3y = 6$
   (D) $4x - 3y = 6$ (E) $4x - 3y = 2$ (F) none of these

3. Find the value of x at the intersection of the line $2x + 3y = 18$ and $x - y = 4$.

   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6 (F) none of these

4. Select the correct statement about the three lines given by the equations:

$$x + y = 2, \quad x = -2, \quad x + 2y = 7.$$

   (A) Two of these lines are parallel.
   (B) Two of these lines have negative slopes, and the slope is undefined for the other line.
   (C) Two of these lines have negative slopes, and the slope is zero for the other line.
   (D) Two of these lines have positive slopes, and the slope is undefined for the other line.
   (E) These lines all go through a single point
   (F) none of the above.

5. Suppose that the cost of renting in-line skates is related to the number of hours the skates are rented by a linear equation. Also, suppose the cost of a 2-hour rental is \$20 and the cost of a 3-hour rental is \$25. Find the cost of a 6-hour rental.

   (A) \$40 (B) \$42 (C) \$44 (D) \$46 (E) \$48 (F) none of these

For each of the augmented matrices in the next three problems, determine which of the following statements is true about the associated system of linear equations:
(A) The system has no solution.
(B) The system has exactly one solution.
(C) The system has exactly two solutions.
(D) The system has infinitely many solutions in which one variable can be selected arbitrarily.
(E) The system has infinitely many solutions in which two variables can be selected arbitrarily.
(F) none of the above.

6. $$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 7 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

7. $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

8. $\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$

9. Determine which of the following matrices are in reduced form.

$$A = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 1 \\ 0 & 1 & 4 & 2 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad B = \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 1 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$$C = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \qquad D = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

(A) C only (B) B and C only (C) A, B and C
(D) B, C and D only (E) A and C only (F) none of these

10. Ralph's Fill Dirt and Croissant Shop in Spencer makes both grand and petit croissants. Each grand requires 1 ounce of flour and 2 ounces of butter, while each petit requires only $\frac{1}{4}$ ounce of flour and $\frac{1}{3}$ ounce of butter. It is the end of the month and Ralph has only 4 ounces of flour and 6 ounces of butter. Ralph calculates how many of each type of croissant he should make in order to use all his ingredients. Determine the number of grand croissants Ralph calculated.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) none of these

Problems 11 and 12 refer to the following matrices

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} -2 \\ 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} \quad E = \begin{pmatrix} 3 & -1 \end{pmatrix}$$

11. Determine which of the following is defined.

(A) $A - DB$ (B) $3B + DC$ (C) $BC + 2D$ (D) AE (E) $BC + E$ (F) none of these

12. Find CA.

(A) $\begin{pmatrix} 1 & 6 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 4 & 3 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 4 & 6 & -1 \end{pmatrix}$
(D) $\begin{pmatrix} 4 & 6 & 1 \end{pmatrix}$ (E) undefined (F) none of these

13. Find the value of x which solves the system of equations:

$$\begin{aligned} 2x + 4y + 10z &= -2 \\ y - 3z &= 4 \\ x + 2y + 6z &= -2 \end{aligned}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) none of these

14. Find all solutions to the following system of equations:

$$\begin{array}{rcrcrcrcl} x & - & y & + & 3z & & & = & 5 \\ 2x & + & y & & & + & 3w & = & 7 \\ -x & + & y & - & 3z & + & 4w & = & 3 \end{array}$$

(A) $x = 2$, $y = -3 + 3z$, z arbitrary, $w = 2$
(B) $x = 2 + z$, $y = -3 + 3z$, z arbitrary, $w = 2$
(C) $x = 3 - z$, $y = -3 + 3z$, z arbitrary, $w = 2$
(D) $x = 2 - z$, $y = -3 + 2z$, z arbitrary, $w = 2$
(E) $x = 2 - z - 2w$, $y = -3 + 2z + w$, z arbitrary, w arbitrary
(F) none of these

15. A certain 3×3 matrix A has as its inverse the matrix

$$A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & -1 & 2 \end{bmatrix}.$$

Determine the value of y where

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5 (F) none of these

16. An economy has two goods: steel and grain. One needs .25 units of steel to produce 1 unit of steel, and .27 units of grain to produce 1 unit of grain. In addition, one needs .2 units of steel to make 1 unit of grain, and .4 units of grain to make one unit of steel. Let x_1 be the number of units of steel produced and x_2 be the number of units of grain produced. Find the technology matrix associated with the corresponding Leontief economic model.

(A) $\begin{pmatrix} .25 & .4 \\ .2 & .27 \end{pmatrix}$ (B) $\begin{pmatrix} .25 & .2 \\ .4 & .27 \end{pmatrix}$ (C) $\begin{pmatrix} .27 & .4 \\ .2 & .25 \end{pmatrix}$

(D) $\begin{pmatrix} .27 & .2 \\ .4 & .25 \end{pmatrix}$ (E) $\begin{pmatrix} .25 & .27 \\ .2 & .4 \end{pmatrix}$ (F) none of these

17. Let

$$A = \begin{pmatrix} .4 & .2 \\ .9 & .2 \end{pmatrix}, \quad D = \begin{pmatrix} 30 \\ 60 \end{pmatrix}, \quad \text{and} \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

be the technology matrix, demand vector, and production schedule for a Leontief economic model. Find x_1.

(A) 80 (B) 90 (C) 100 (D) 110 (E) 120 (F) none of these

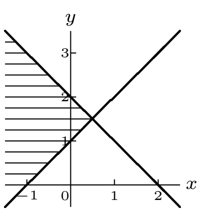
18. An enterprising student plans to make and sell novelty T-shirts at the Little 500. A simpler style requires 1 ounce of ink and 20 minutes of labor for each shirt, while a more elaborate style requires 3 ounces of ink and 70 minutes to make each shirt. The student has $\frac{1}{2}$ gallon of ink and can spend at most 20 hours making the T-shirts. The profit on the simpler style is \$3 per shirt, and the profit on the more elaborate shirt is \$5. Let x represent the number of simpler style shirts and y the number of more elaborate style shirts. Formulate a linear programming problem to maximize the profit. (Recall that there are 128 ounces in a gallon.)

    Maximize $3x + 5y$ *subject to*

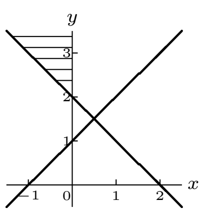
    (A) $x + 3y \leq 128,\ 20x + 70y \leq 20,\ x \geq 0,\ y \geq 0$
    (B) $x + 3y \leq 128,\ 20x + 70y \leq 1200,\ x \geq 0,\ y \geq 0$
    (C) $3x + y \leq 64,\ 20x + 70y \leq 1200,\ x \geq 0,\ y \geq 0$
    (D) $3x + y \leq 64,\ 70x + 20y \leq 20,\ x \geq 0,\ y \geq 0$
    (E) $x + 3y \leq 64,\ 20x + 70y \leq 1200,\ x \geq 0,\ y \geq 0$
    (F) none of these

19. Determine the feasible set given by the constraints

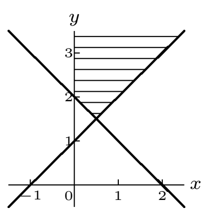
$$x + y \geq 2, \quad -x + y \geq 1, \quad x \geq 0, \quad y \geq 0$$



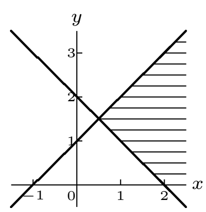
(A)



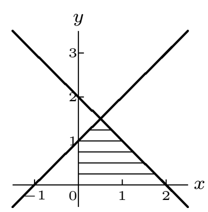
(B)



(C)



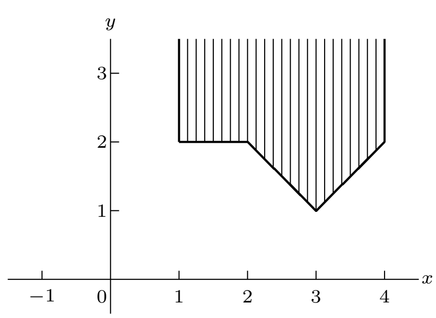
(D)



(E)

(F) none of these

20. Find the *minimum* of $x + y$ on the feasible set shown below.



(A) 3 (B) 4 (C) 5 (D) 6 (E) there is no minimum (F) none of these

FINAL EXAMS

Sample exams included in textbook, Appendix D

ANSWERS FOR SAMPLE EXAMS 1 AND 3

Sample Exam 1, Version 1

1. C	5. E	9. C	13. A	17. C
2. B	6. B	10. C	14. C	18. B
3. C	7. B	11. C	15. A	19. C
4. E	8. C	12. D	16. A	20. B

Sample Exam 1, Version 2

1. B	5. A	9. A	13. C
2. A	6. B	10. E	14. True
3. C	7. D	11. D	15. False
4. A	8. A	12. D	16. False

Sample Exam 3, Version 1

Part 1.

1. C	4. A	7. C	10. A	13. C
2. B	5. C	8. B	11. B	
3. D	6. D	9. C	12. A	

Part 2.

1. To maximize profit, Barbara's should produce 25 Colonial baskets and 75 Southwest baskets each week.

2. First, write this system as an augmented matrix $\left[\begin{array}{rrr|r} 2 & -5 & 0 & 0 \\ 1 & -3 & -1 & -1 \\ -1 & 2 & -1 & -1 \end{array}\right]$. Then perform Gaussian elimination to get $\left[\begin{array}{rrr|r} 1 & 0 & 5 & 5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$. So $x + 5z = 5$ and $y + 2z = 2$, and the answer is $x = 5 - 5z$, $y = 2 - 2z$, and z is arbitrary.

3. To find the inverse, reduce the augmented matrix $\left[\begin{array}{rrr|rrr} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 4 & 0 & 0 & 1 \end{array}\right]$. The inverse $\mathbf{A}^{-1}$ is the matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ -2 & -1 & 1 \end{bmatrix}$.

Sample Exam 3, Version 2

1. B	5. A	9. B	13. B	17. E
2. D	6. E	10. A	14. D	18. E
3. E	7. A	11. C	15. E	19. C
4. B	8. D	12. D	16. B	20. A