Practice Exam 2

1. Consider a random variable X with probability distribution function

$$f(x) = \begin{cases} 1/2 & \text{for } 0 < x < 1 \\ 1/2 & \text{for } 2 < x < 3 \\ 0 & \text{elsewhere} \end{cases}$$

(A) Sketch f(x).
(B) Find the cumulative distribution function, $F(x)$, and sketch it.
(C) Find EX.
(D) Find var(X).

2. Let P be uniform on $[0, 1/2]$. Given $P = p$, let X be Bernoulli with parameter p. Find the conditional distribution of P given X.

3. (A) Find the density of $Y = \sqrt{X}$ where X is exponential with rate 1.
   (B) How would you use random uniform random variables to generate a collection of random variables from the density of Y from part (A)?

4. Give an example of two uncorrelated random variables which are not independent. Uncorrelated means that $E[(X - EX)(Y - EY)] = 0$.

5. Suppose you choose two numbers x and y, independently at random from the interval $[0, 1]$. Given that their sum lies in the interval $[0, 1]$, find the probability that

(A) $|x - y| < 1$.

(B) $xy < 1/2$.

(C) $\max\{x, y\} < 1/2$.

(D) $x^2 + y^2 < 1/4$.

(E) $x > y$.

6. Find the expected value and variance of the geometric distribution. Hints:

(a) $1 + 2x + 3x^2 + 4x^3 + \cdots = \dfrac{d}{dx}\left(1 + x + x^2 + x^3 + x^4 + \ldots\right)$
(b) Consider $E[X(X - 1)]$.