Homework Problem Set 3

(3.1.6) 1. In arranging people around a circular table, we take into account their seats relative to each other, not the actual position of any one person. Show that n people can be arranged around a circular table in $(n-1)!$ ways.

2. In how many ways can 10 symmetrical keys be arranged on a key ring?

(3.1.13) 3. A certain state has license plates showing three numbers and three letters. How many different license plates are possible

(a) if the numbers must come before the letters?

(b) if there is no restriction on where the letters and numbers appear?

(3.1.16) 4. Prove that at least two people in Atlanta, Georgia, have the same initials, assuming no one has more than four initials.

5. What is the probability that 3 random people will have the same birthday (month and day, assuming 365 days in a year)?

(3.2.6) 6. Charles claims that he can distinguish between beer and ale 75 percent of the time. Ruth bets that he cannot and, in fact, just guesses. To settle this, a bet is made: Charles is to be given ten small glasses, each having been filled with beer or ale, chosen by tossing a fair coin. He wins the bet if he gets seven or more correct. Find the probability that Charles wins if he has the ability that he claims. Find the probability that Ruth wins if Charles is guessing.

(3.2.12) 7. A poker hand is a set of 5 cards randomly chosen from a deck of 52 cards. Find the probability of a

(a) royal flush (ten, jack, queen, king, ace in a single suit).

(b) straight flush (five in a sequence in a single suit, but not a royal flush).

(c) four of a kind (four cards of the same face value).

(d) full house (one pair and one triple, each of the same face value).

(e) flush (five cards in a single suit but not a straight or royal flush).

(f) straight (five cards in a sequence, not all the same suit). (Note that in straights, an ace counts high or low.)

(3.2.22) 8. How many ways can six indistinguishable letters be put in three mail boxes? *Hint*: One representation of this is given by a sequence |LL|L|LLL| where the |'s represent the partitions for the boxes and the L's the letters. Any possible way can be so described. Note that we need two bars at the ends and the remaining two bars and the six L's can be put in any order.

(3.2.23) 9. Using the method for the hint in Exercise 3.2.22, show that r indistinguishable objects can be put in n boxes in

$$\binom{n+r-1}{n-1} = \binom{n+r-1}{r}$$

different ways.

(3.2.29) 10. A drug is assumed to be effective with an unknown probability p. To estimate p the drug is given to n patients. It is found to be effective for m patients. The *method of maximum likelihood* for estimating p states that we should choose the value for p that gives the highest probability of getting what we got on the experiment. Assuming that the experiment can be considered as a Bernoulli trials process with probability p for success, show that the maximum likelihood estimate for p is the proportion m/n of successes.