Homework Problem Set 2

(2.2.1) 1. Suppose you choose *at random* a real number X from the interval $[2, 10]$.

(a) Find the density function $f(x)$ and the probability of an event E for this experiment, where E is a subinterval $[a, b]$ of $[2, 10]$.

(b) From (a), find the probability that $X > 5$, that $5 < X < 7$, and that $X^2 - 12X + 35 > 0$.

(2.2.2) 2. Suppose you choose a real number X from the interval $[2, 10]$ with a density function of the form

$$f(x) = Cx \ ,$$

where C is a constant.

(a) Find C.

(b) Find $P(E)$, where $E = [a, b]$ is a subinterval of $[2, 10]$.

(c) Find $P(X > 5)$, $P(X < 7)$, and $P(X^2 - 12X + 35 > 0)$.

(2.2.3) 3. Same as Exercise 2.2.2, but suppose

$$f(x) = \frac{C}{x} \ .$$

(2.2.4) 4. Suppose you throw a dart at a circular target of radius 10 inches. Assuming that you hit the target and that the coordinates of the outcomes are chosen at random, find the probability that the dart falls

(a) within 2 inches of the center.

(b) within 2 inches of the rim.

(c) within the first quadrant of the target.

(d) within the first quadrant and within 2 inches of the rim.

HPS 5. Let a point be chosen uniformly from the interior of a triangle with base 10 units and height 6 units from the base. Let X denote the distance from the point chosen to the base. Find the density function for X.

(2.2.6) 6. Assume that a new light bulb will burn out after t hours, where t is chosen from $[0, \infty)$ with an exponential density

$$f(t) = \lambda e^{-\lambda t} \ .$$

In this context, λ is often called the *failure rate* of the bulb.

(a) Assume that $\lambda = 0.01$, and find the probability that the bulb will *not* burn out before T hours. This probability is often called the *reliability* of the bulb.

(b) For what T is the reliability of the bulb $= 1/2$?

(2.2.7) 7. Choose a number B *at random* from the interval $[0, 1]$ with uniform density. Find the probability that

(a) $1/3 < B < 2/3$.

(b) $|B - 1/2| \le 1/4$.

(c) $B < 1/4$ or $1 - B < 1/4$.

(d) $3B^2 < B$.

(2.2.8) 8. Choose independently two numbers B and C *at random* from the interval $[0, 1]$ with uniform density. Note that the point (B, C) is then chosen *at random* in the unit square. Find the probability that

(a) $B + C < 1/2$.

(b) $BC < 1/2$.

(c) $|B - C| < 1/2$.

(d) $\max\{B, C\} < 1/2$.

(e) $\min\{B, C\} < 1/2$.

(f) $B < 1/2$ and $1 - C < 1/2$.

(g) conditions (c) and (f) both hold.

(h) $B^2 + C^2 \le 1/2$.

(i) $(B - 1/2)^2 + (C - 1/2)^2 < 1/4$.

(2.2.12) 9. Take a stick of unit length and break it into three pieces, choosing the break points at random. (The break points are assumed to be chosen simultaneously.) What is the probability that the three pieces can be used to form a triangle? *Hint*: The sum of the lengths of any two pieces must exceed the length of the third, so each piece must have length $< 1/2$. Now use Exercise 2.2.8(g).

(2.2.14) 10. Choose independently two numbers B and C *at random* from the interval $[-1, 1]$ with uniform distribution, and consider the quadratic equation

$$x^2 + Bx + C = 0 .$$

Find the probability that the roots of this equation

(a) are both real.

(b) are both positive.

Hints: (a) requires $0 \le B^2 - 4C$, (b) requires $0 \le B^2 - 4C$, $B \le 0$, $0 \le C$.

HPS = Hoel, Port, and Stone