Exam 2

Name: ______________________________

Your best 5 out of 6 problems will form the basis of your grade on this exam. If you work all 6 problems essentially correctly, that will also be noted. For some problems on this exam, it is useful to know that $-\ln x$ is a density on $(0,1)$. You can verify this by using integration by parts and by determining that $\lim_{x\to 0} -x\ln(x) = \lim_{x\to\infty} \frac{\ln(x)}{x} = 0$.

1: Consider the joint distribution of two random variables:

$$f_{X,Y}(x,y) = -\ln(x^{2y}) \text{ for } 0 < x < 1 \text{ and } 0 < y < 1.$$

(A) Are X and Y independent?

(B) Find the marginal distribution $f_X(x)$. What is EX?

(C) Find the marginal distribution $f_Y(y)$. What is EY?

2: Let X and Y be independent, identically distributed exponential random variables with rate 1. Find the density for $U = X/(X+Y)$. Be careful to specify where this density lives.

3: Let X and Y be independent, identically distributed, uniform $(0, 1)$ random variables. Find the joint density for $U = XY$ and $V = X/Y$. Be careful to specify where this density lives.

4: (A) State Markov's inequality.

(B) Let X be a geometric distribution with $p = 1/2$. What does Markov's inequality say about the probability X is at least k for $k = 2, 4$, and 10? What are the actual probabilities?

5: Find the joint distribution of the minimum and the maximum of three independent exponential random variables with rate 1. That is, find the joint distribution of $X_{(1)}$ and $X_{(3)}$.

6: Let X be a random variable on $(0, 1)$ with density $f_X(x) = -\ln(x)$. Describe how you could use the rejection method to sample from X. (Your bounding function M will not be bounded. It should be simple enough for you to sample from easily. There are continuous functions that will work. Or you can use a clever step function.)